國立臺北科技大學 101 學年度碩士班招生考試

系所組別：2210 電腦與通訊研究所甲組

第一節　工程數學　試題

> **注意事項：**
> 1. 本試題共七題，配分共 100 分。
> 2. 請標明大題、子題編號作答，不必抄題。
> 3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、

(a). Find the determinant of the matrix A given below. (5%)

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 3 & -5 \\ 3 & 1 & 3 \end{bmatrix}$$

(b). Determine whether the column vectors of A, i.e., $(4, 2, 3)^T$, $(2, 3, 1)^T$, and $(2, -5, 3)^T$, are linearly independent. (5%)

二、

Determine the least squares solution to $Ax = b$, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 0 & -1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$ (10%)

三、

The matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 6 \\ -2 & 2 & 7 \end{bmatrix}$ has a LU-factorization, i.e., $A = LU$. Find the matrix L and U,

where L is a lower triangular matrix with its diagonal entries equal to 1, and U is an upper triangular matrix. (15%)

四、

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 3 \\ 1 & 1 & -1 \end{bmatrix}$.

(a). Find the eigenvalues and the corresponding eigenvectors of matrix A. (10%)

(b). Is matrix A diagonalizable? That is, can we find a nonsingular matrix S and a diagonal matrix D such that $S^{-1}AS = D$? If the answer is "Yes", find the resulted diagonal matrix D and the nonsingular matrix S that diagonalizes A. On the other hand, give the reason if your answer is "No". (10%)
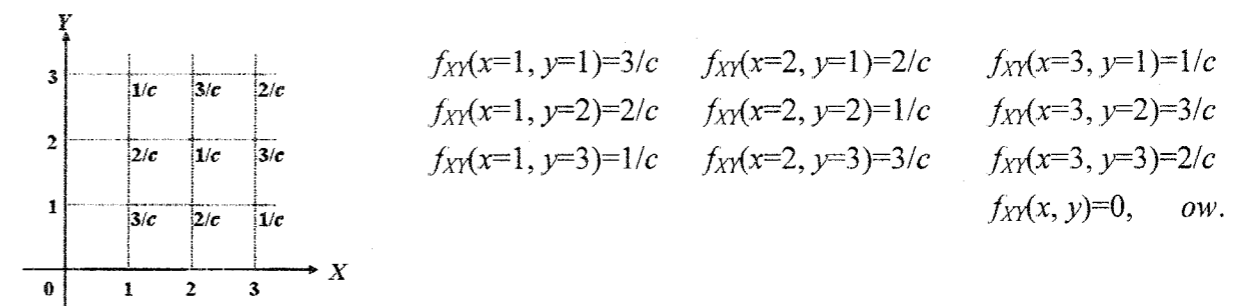
五、

A cell phone may come from any one of the two factories A and B with probabilities $P_A = 0.6$ and $P_B = 0.4$. The probabilities that the cell phone will be defective during the manufacturing process are 0.04 and 0.02, respectively.

(a). Find the probability that a randomly chosen cell phone will be defective. (5%)

(b). If the chosen cell phone is defective, what is the probability that this cell phone comes from factory A? (10%)

六、

The joint probability density function $f_{XY}(x, y)$ of two discrete random variables X and Y is given below

$f_{XY}(x=1, y=1)=3/c$	$f_{XY}(x=2, y=1)=2/c$	$f_{XY}(x=3, y=1)=1/c$
$f_{XY}(x=1, y=2)=2/c$	$f_{XY}(x=2, y=2)=1/c$	$f_{XY}(x=3, y=2)=3/c$
$f_{XY}(x=1, y=3)=1/c$	$f_{XY}(x=2, y=3)=3/c$	$f_{XY}(x=3, y=3)=2/c$
		$f_{XY}(x, y)=0, \quad ow.$

(a). Find the value of c. (4%)

(b). Find the marginal p.d.f. of X and Y, i.e., to find $f_X(x)$ and $f_Y(y)$. (6%)

(c). Find the mean and variance of X and Y. (4%)

(d). Evaluate the correlation coefficient of X and Y, i.e., to find $\rho(X, Y)$. (4%)

(e). Are the two random variables X and Y independent? Prove your answer. (2%)

七、

A machine is used to check the defectiveness of a product, i.e., to check if the product is good or defective. Due to some reasons the machine can have a probability $q=0.1$ of wrong decision. To conquer this problem, each product is checked by this machine **three** times to reduce the probability of making wrong decision. Therefore, a product will be regarded as defective if more than two times is detected as defective. Please determine the probability of **wrong decision** with this procedure.

(10%)