國立臺北科技大學 101 學年度碩士班招生考試

系所組別：2110、2120、2130 電機工程系碩士班甲、乙、丙組

第二節　工程數學　試題

注意事項：

1. 本試題共 6 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (15%) Find the general solution of the nonlinear differential equation:

$$y' + y\ln x^3 = 3x^{-x}e^x y^{2/3}.$$

2. (15%) Find the general solution of the Cauchy-Euler differential equation:

$$(x-1)^2 y'' + (x-1)y' + y = x, \quad x > 1$$

3. (20%) Find the solution of the integrodifferential equation:

$$y' + 5y + 6\int_0^x y(\tau)d\tau = f(x);\ y(0) = 1.$$

where $f(x) = \begin{cases} 1 & x < 2 \\ 0 & 2 \le x < 4 \\ 1 & x \ge 4 \end{cases}$

4. Let $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 6 \\ 0 \\ -2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -1 \\ -3 \\ 3 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 3 \\ 9 \\ -2 \\ -3 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 0 \\ 4 \\ -5 \\ 1 \end{bmatrix}$. The subspace W of $\mathbf{R}^4$ is spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$.

(5%) (a) Find a basis for the subspace W.

(10%) (b) Find an orthogonal basis for the subspace W.

(5%) (c) Find an orthonormal basis for the subspace W.

5. (15%) Let $A = \begin{bmatrix} 4 & -3 \\ 0 & 1 \end{bmatrix}$. Evaluate $A^{1/2}$.

6. Let $B = \begin{bmatrix} -5 & -8 & 0 & 3 \\ 0 & 5 & 0 & -6 \\ 5 & -7 & 2 & 2 \\ 0 & 3 & 0 & -4 \end{bmatrix}$.

Mark the following statements True or False. Justify each answer.

(5%) (a) B is an invertible matrix.

(5%) (b) The linear transformation $\mathbf{x} \mapsto B\mathbf{x}$ for any vector $\mathbf{x} \in \mathbf{R}^4$ is one-to-one.

(5%) (c) Dimension of the null space of B is n.