

國立臺北科技大學 101 學年度碩士班招生考試

系所組別：1511、1512 自動化科技研究所 甲組

第一節 工程數學 試題

第一頁 共一頁

注意事項：

1. 本試題共六題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (15%) For vector space

- (1)(5%) Let v_1, v_2, \dots, v_m are the vectors of vector space V . Explain that linearly dependent and linearly independent for v_1, v_2, \dots, v_m .
- (2)(5%) Are $\cos(x)$ and $\sin(x)$ linearly dependent or linearly independent? Why?
- (3)(5%) Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $C = \begin{bmatrix} -1 & -2 \\ -3 & -5 \end{bmatrix}$, $D = \begin{bmatrix} -1 & -2 \\ 0 & -2 \end{bmatrix}$. Are A, B, C, D, linearly dependent or linearly independent? Why?

2. (15%) If u and v are any two vectors in an inner product space V , then

$$|\langle u, v \rangle| \leq \|u\| \|v\|$$

Equality holds if and only if u and v are linearly dependent. Please prove this Cauchy-Schwarz theorem.

3. (20%) For each matrix, find the characteristic equation, and the eigenvalues and associated eigenvectors

(1)(10%)

$$\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

(2)(10%)

$$\begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$$

4. (20%) Shows the following operations are isotropic (invariant to rotation).

(1)(10%) The Laplacian operation

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

(2)(10%) The magnitude of the gradient

$$|\nabla f| = \text{mag}(\nabla f) = [G_x^2 + G_y^2]^{1/2} = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

5. (15%) Solve the differential equation by Laplace transform.

$$y'' + y = \delta(x-1) \quad 0 < x < 2$$

The boundary conditions are $y(0)=0$ and $y(2)=0$.

6. (15%) Solve the differential equations

$$mx_1'' + (k + k_1)x_1 - k_1x_2 = 0$$

$$mx_2'' + (k + k_1)x_2 - k_1x_1 = 0$$

where m , k , and k_1 are positive real number.