國立臺北科技大學 101 學年度碩士班招生考試

系所組別：1410、1420、1431、1432

能源與冷凍空調工程系碩士班甲、乙、丙組

第一節　工程數學　試題

第一頁　共一頁

注意事項：

1. 本試題共五題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. **(30%)** Find the general solutions of the following equations:

(1) (10%) $y' + \frac{1}{x} y = \cos x$

(2) (10%) $y' + x^2 y = x^2 y^3$

(3) (10%) $(2x+1)^2 y'' - 2(2x+1)y' - 12y = 2x \quad (2x+1>0)$

2. **(20%)** Please solve the following problems by the method of Laplace transform:

(1) (10%) $\begin{cases} \frac{dx}{dt} = 4x + y \\ \frac{dy}{dt} = 3x + 2y \end{cases}, \quad y(0) = y'(0) = 1$

(2) (10%) $y'' + 5y' + 4y = 0, \quad y(0) = 1, \quad y(1) = 0$

3. **(20%)** If a differential equation, $\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0$, is subjected to the following boundary conditions,

$$\theta(0) = \theta_0, \quad \left.\frac{d\theta}{dx}\right|_{x=L} = 0 \qquad \text{(a)}$$

$$\theta(0) = \theta_0, \quad \theta(L) = \theta_L \qquad \text{(b)}$$

please prove that the solutions of the above differential equation associated with the boundary conditions (a) and (b) are as follows:

(1) (8%) $\frac{\theta}{\theta_0} = \frac{\cosh m(L-x)}{\cosh mL}$ for boundary conditions (a)

(2) (12%) $\frac{\theta}{\theta_0} = \frac{(\theta_L/\theta_0)\sinh mx + \sinh m(L-x)}{\sinh mL}$ for boundary conditions (b)

Note: *m, h, k, L,* θ_0, and θ_L are real constants.

4. **(15%)** Solve the initial value problem by means of the method of the undetermined coefficients.

$$y'' + 2y' + y = e^{-x}, \quad y(0) = -1, \quad y''(0) = 1$$

5. **(15%)** The *Bessel function of the first kind of order v* is given by

$$J_v(x) = x^v \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+v} m! \Gamma(m+v+1)}$$

The function $I_v(x) = i^{-v} J_v(ix)$, $i = \sqrt{-1}$, is called the *modified Bessel function of the first kind of order v*. Show that $I_v(x)$ is a solution of the differential equation

$$x^2 y'' + xy' - (x^2 + v^2) y = 0$$

and has the representation

$$I_v(x) = x^v \sum_{m=0}^{\infty} \frac{x^{2m+v}}{2^{2m+v} m! \Gamma(m+v+1)}$$