國立臺北科技大學 101 學年度碩士班招生考試

系所組別：1310、1320、1330

車輛工程系碩士班甲、乙、丙組

第二節　工程數學　試題

第一頁　共一頁

注意事項：
1. 本試題共十題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、Consider the first-order differential equation $y' = y^2 + 3y + 2$.

1. (5%) Find the general solution.
2. (5%) Find the solution for the initial condition $y(0) = -2$.

二、(10%) Solve the initial value problem

$y'' + 4y' + 5y = f(x); y(\pi) = 1, y'(\pi) = 2$.

三、Consider the second-order differential equation $y'' + 4y' + 5y = f(x)$.

1. (5%) Find the particular solution when $f(x) = x$.
2. (5%) Find the particular solution when $f(x) = \sin(x)$.

四、(10%) If the Laplace transform of $f(t)$ is $\dfrac{1}{(s+1)(1-e^{-2s})}$, find $f(3)$ and the maximum value of $f(t)$ when $t > 0$. Hint: $1 + r + r^2 + \cdots = 1/(1-r)$

五、(10%) Solve the initial value problem

$y' + y + \int_0^t e^{-(t-\tau)} y(\tau) d\tau = 0; y(0) = 2$.

六、Consider the system $\begin{cases} 5x_1 + 4x_2 = \lambda x_1 \\ 4x_1 - x_2 = \lambda x_2 \end{cases}$ where x_1, x_2 are unknowns.

1. (5%) Find λ such that the system has nonzero solutions.
2. (5%) Find the nonzero solutions for each λ.

七、(10%) Consider the system

$$\vec{x}(n+1) = \mathbf{A}\vec{x}(n) \quad \text{where} \quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \mathbf{A} = \begin{bmatrix} 5 & 4 \\ 4 & -1 \end{bmatrix}, n = 0, 1, 2, \cdots.$$

If $x_1(0) = x_2(0) = 5$, find $x_1(n)$ and $x_2(n)$ represented by numbers and n.

八、Every vector in a vector space can be represented by

$$\alpha[1 \ \ 0 \ \ 1 \ \ 2] + \beta[1 \ \ -1 \ \ 2 \ \ 1] + \gamma[1 \ \ 1 \ \ 0 \ \ 3] + \delta[2 \ \ 1 \ \ 1 \ \ 5].$$

1. (5%) Find the dimension of this vector space.
2. (5%) Find a basis of the solution space of the system $\mathbf{A}\vec{x} = 0$ where

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \mathbf{A} = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & -1 & 2 & 1 \\ 1 & 1 & 0 & 3 \\ 2 & 1 & 1 & 5 \end{bmatrix}.$$

九、Let α and β be two distinct eigenvalues of a symmetry matrix $\mathbf{A}$. Let $\vec{u}$ and $\vec{v}$ be the eigenvectors corresponding to α and β, respectively.

1. (5%) Prove that $\vec{u}$ and $\vec{v}$ are orthogonal. Hint: prove that $\vec{u}^T \vec{v} = 0$.
2. (5%) Prove that all eigenvalues of $\mathbf{A}$ are real. Hint: prove that $\bar{\alpha} = \alpha$ where $\bar{\alpha}$ is the complex conjugate of α.

十、(10%) Evaluate the determinant $\begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & -1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$.