國立勤益科技大學 101 學年度研究所碩士班招生筆試試題卷

所別： 工業工程與管理　　　　　　　　組別：作業研究

科目： 作業研究

准考證號碼：□□□□□□□□（考生自填）

考生注意事項：

一、考試時間 100 分鐘。

試題一：***True/False***〈Each 5 points, total 30 points〉

Indicate by “O” = “true” or “X” =”false.”

__ 1. The “minimum ratio test” is used to determine the pivot row in the simplex method.

__ 2. A “pivot” in the simplex method corresponds to a move from one corner point of the feasible region to another.

__ 3. Adding constraints to an LP may improve the optimal objective function value.

__ 4. If an artificial variable is nonzero in the optimal solution of an LP problem, then the problem has no feasible solution.

__ 5. The optimal values of the primal and dual LP problems, if they exist, must be equal.

__ 6. The two-phase method solves for the dual variables in Phase I, and then solves for the primal variables in Phase II.

試題二：Matrix multiplication.〈Each 3 points, total 30 points〉

1. $\begin{bmatrix} 1 & -1 & 6 \\ 2 & 0 & -1 \\ 3 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

2. The inverse matrix of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

試題三：LP Duality.〈Each 2 points, total 42 points〉

Consider the following **primal** LP problem:

Maximize $z = 60x_1 + 30x_2 + 20x_3$

Subject to

$8x_1 + 6x_2 + x_3 \le 48$

$4x_1 + 2x_2 + 1.5x_3 \le 20$

$2x_1 + 1.5x_2 + 0.5x_3 \le 8$

$x_1, x_2, x_3 \ge 0$

1. Write the **dual** of the above problem, filling the blanks with numbers and the boxes with ≥, =, ≤ or "U"*(unrestricted in sign)*.

Minimize $w =$ ___ $y_1 +$ ___ $y_2 +$ ___ y_3

Subject to ___ $y_1 +$ ___ $y_2 +$ ___ y_3 □ ___

___ $y_1 +$ ___ $y_2 +$ ___ y_3 □ ___

___ $y_1 +$ ___ $y_2 +$ ___ y_3 □ ___

$y_1 \geq 0, y_2 \geq 0, y_3 \geq 0$

2. Given the optimal solution to the primal problem is $z = 280$, $x_1 = 2$, $x_2 = 0$, $x_3 = 8$, use *complementary slackness* property to solve the dual problem, i.e., $y_1 =$ ___, $y_2 =$ ___, $y_3 =$___

試題四：〈每小題 15 分，總分 30 分〉

某公司兩種產品的利潤函數如下：$P = -x^2 - 2y^2 + xy - 30$。

1. 若客戶訂單上要求為：$x + y = 34$，利潤最大時 x、y 各為多少？
2. 若客戶訂單上要求 $x + y = 35$，利潤大約變化多少？

試題五：〈每小題 10 分，總分 30 分〉

甲、乙二人對局，甲方之償付矩陣如下表：

甲策略 \ 乙策略	B1	B2
A1	3	-4
A2	-3	0

1. 期望報酬最大時，甲方混合策略為何？
2. 期望報酬最大時，乙方混合策略為何？
3. 本對局對誰較有利？期望報酬為何？

試題六：〈每小題 19 分，總分 38 分〉

汽車以平均每小時90輛之Poisson分配到達收費站，通過關卡之平均時間為36秒。請問：

1.系統中的等候線長度為何？

2.若加入新裝置使平均服務時間降為30秒，則新系統收費站之空閒時間比率多少？

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<u>X</u> 6. The two-phase method solves for the dual variables in Phase I, and then solves for the primal variables in Phase II.

試題二：Matrix multiplication.〈Each 3 points, total 30 points〉

1. $\begin{bmatrix} 1 & -1 & 6 \\ 2 & 0 & -1 \\ 3 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 9 & 1 \\ 5 & 4 \\ 11 & 7 \end{bmatrix}$

2. The inverse matrix of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$

試題三：LP Duality.〈Each 2 points, total 42 points〉

1. Minimize $\mathbf{w = \underline{48}y_1 + \underline{20}y_2 + \underline{8}y_3}$

Subject to
$$\mathbf{\underline{8}y_1 + \underline{4}y_2 + \underline{2}y_3 \boxed{\geq} \underline{60}}$$
$$\mathbf{\underline{6}y_1 + \underline{2}y_2 + \underline{1.5}y_3 \boxed{\geq} \underline{30}}$$
$$\mathbf{\underline{1}y_1 + \underline{1.5}y_2 + \underline{0.5}y_3 \boxed{\geq} \underline{20}}$$
$$y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0$$

2. Given the optimal solution to the primal problem is $z = 280$, $x_1 = 2$, $x_2 = 0$, $x_3 = 8$, use *complementary slackness* property to solve the dual problem, i.e., $\mathbf{y_1 = \underline{0}, y_2 = \underline{10}, y_3 = \underline{10}}$.

試題四：〈每小題 15 分，總分 30 分〉

1. L(x, y, λ)= $-x^2-2y^2+xy-30+\lambda(x+y-34)$

$$\frac{\partial L}{\partial x}=-2x+y+\lambda=0............(1)$$

$$\frac{\partial L}{\partial y}=-4y+x+\lambda=0............(2)$$

$$\frac{\partial L}{\partial \lambda}=x+y-34=0................(3)$$

(1)-(2)得$-3x+5y=0............(4)$

(3)*3+(4)得$8y=102$ ===> $y=102/8$

代入(3)得 $x=170/8$

2. 由x, y的值及(1)式，得$\lambda=2x-y=(340/8)-(102/8)=238/8$

所以當$x+y=35$時目標值約增加 238/8

試題五：〈每小題 10 分，總分 30 分〉

1.設甲方混合策略為$(x, 1-x)$，

若已方採 B1 策略，甲的期望報酬為 E1=$3x-3(1-x)=6x-3$

若已方採 B2 策略，甲的期望報酬為 E2=$-4x+0(1-x)=-4x$

當甲方採最佳混合策略時，不管乙方採何策略，甲方之期望報酬必相等。

$$E1=E2 ===> 6x-3=-4x =====> x=0.3,\quad 1-x=0.7$$

2.同理，乙方之最佳混合策略$(y, 1-y)$如下

$$3y-4(1-y)=-3y+0(1-y) ========> y=0.4,\quad 1-y=0.6$$

3.本賽局對 B 較有利，B 贏的期望報酬為 1.2。

$$3(0.3)-3(0.7)=-4(0.3)+0(0.7)=3(0.4)-4(0.6)=-3(0.4)+0(0.6)=-1.2$$

試題六：〈每小題 19 分，總分 38 分〉

1. $\rho=\frac{\lambda}{\mu}=\frac{90}{\frac{3600}{36}}=0.90$

$$L=\frac{\rho}{1-\rho}=\frac{0.90}{0.10}=9$$

2. $\rho=\frac{\lambda}{\mu}=\frac{90}{\frac{3600}{30}}=0.75$

$$p_0=1-\rho=1-0.75=0.25$$