國立雲林科技大學
101 學年度碩士班暨碩士在職專班招生考試試題
系所: 財金系
科目: 統計學(4)

1. (10 points) Records of a company show that 20% of the employees have only a high school diploma; 70% have bachelor degrees; and 10% have graduate degrees. Of those with only a high school diploma, 10% hold management positions; whereas, of those having bachelor degrees, 40% hold management positions. Finally, 80% of the employees who have graduate degrees hold management positions.

a. What percentage of employees holds management positions?
b. Given that a person holds a management position, what is the probability that she/he has a graduate degree?

2. (10 points) In a survey conducted by the Gallup Sports Organization, respondents were asked, "What is your favorite sport to watch?" Football and basketball ranked number one and two in terms of preference. Assume that in a group of 10 individuals, seven preferred football and three preferred basketball. A random sample of three of these individuals is selected.
a. What is the probability that exactly two preferred football?
b. What is the probability that the majority (either two or three) preferred football?

3. (10 points) Consider the following results for independent samples taken from two populations.

Sample 1	Sample 2
$n_1 = 400$	$n_2 = 300$
$\overline{p_1} = 0.48$	$\overline{p_2} = 0.36$

a. Develop a 95% confidence interval for the difference between the two population proportions.

4. (10 points) If the joint probability density of X and Y is given by

$$f(x,y) = f(x) = \begin{cases} \frac{1}{4}(2x + y), & \text{for } 0 < x < 1, 0 < y < 2 \\ 0 & \text{, elsewhere} \end{cases}$$

find
a. the marginal density of Y;
b. the conditional density of X given Y=1.

5. (10 points) If a random variable X has a uniform density with the parameters α and β, find its distribution function.

6. (25 points) In the simple regression Model, $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$，i = 1,2,3, ... , n. X is control variable. We utility the method of least square to estimate β_0, β_1 with $E(\varepsilon_i) = 0$, $Var(\varepsilon_i) = \sigma^2$, and ε_i is uncorrelated

a. Please show that $\frac{\sum_{i=1}^{n} \hat{Y}_i}{n} = \bar{Y}$ (5 points)

b. Please show that $\sum_{i=1}^{n} e_i X_i = 0$ (5 points)

c. Please show that $E(MSE) = \sigma^2$, $MSE = \frac{SSE}{n-2}$ (5 points)

d. Please show that $Cov\left(\bar{Y}, \hat{\beta}_1\right) = 0$ (5 points)

e. Please show that $Cov\left(\hat{\beta}_0, \hat{\beta}_1\right) = \frac{-\bar{X}\sigma^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}$ (5 points)

7. (25 points) Suppose that the least square regression line $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ for these data. X is rate of market return (R_m) , Y is that the rate of stock return minus riskless rate ($R_i - R_f$) with $E(\varepsilon_i) = 0$, $Var(\varepsilon_i) = \sigma^2$. The coefficient of determination is 0.9025.

X	60	30	40	-30	20	-40	0	-10	-50	-20
Y	100	70	80	10	60	20	50	30	20	10

$\sum_{i=1}^{n} X_i = 0$，$\sum_{i=1}^{n} Y_i = 450$，$\sum_{i=1}^{n} X_i^2 = 12000$，

$\sum_{i=1}^{n} Y_i^2 = 29300$，$\sum_{i=1}^{n} X_i Y_i = 9900$

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Table1 Results of ANOVA

Sources	DF	SS (sum of square)	MS (mean of square)	F value	P value of F
regression	1	(A)	(C)	(E)	0.00003
error	8	(B)	(D)		
total	9	9050			

Table2 The Empirical Results of Regression Model

	Coefficient	S.E.	t value	p value of t
β_0	(F)	3.321333	(H)	0.00000
β_1	(G)	0.095879	(I)	0.03

a. Please fill out (A) to (E) of table 1 (10 points)

b. Please fill out (F) to (I) of table 2 (8 points)

c. Does the model satisfy goodness of fit at $\alpha = 0.01$ of significant level? Why?(2 points)

d. Compute 90% confidence limits for Y when X is equal to 50.(only show your equation) (3 points)

e. With regarding the empirical results of Table 2, is the capital asset pricing model supported？Why? (2 points)