# 國立雲林科技大學
101 學年度碩士班暨碩士在職專班招生考試試題

系所：工管所
科目：作業研究

1. Consider a supply chain consisting of two firms: a manufacturer and a carrier. To satisfy a known demand, the manufacturer chooses its production amounts for four plants that produce the same product but have different requirements of resources and different margins, as summarized in Table 1(a). The total demand of the market is 100 units, and overproduction is not allowed. Once the production is finished, the manufacturer will deliver products to the market arranged by a carrier. The carrier considers each plant as a node and decides its distribution plan after the manufacturer's production plan is determined. Table 1(b) summarizes the information of carrier's unit shipping cost of each path and path capacities. Each of the firms makes decisions by optimizing the individual profit or cost. (The manufacturer's and carrier's problems satisfy the properties required for LP models.)

   (a) What properties are required for formulating the firms' problems as LP models? If the market demand follows a uniform distribution, what property is violated? (10%)

   (b) Find the optimal production quantities of the four plants and profit obtained by the manufacturer. (10%)

   (c) From part (2), find the shadow price of each resource, and interpret that increasing what resource provides the greatest profit improvement? (10%)

   (d) Solve the carrier's problem as a minimum-cost flow problem. Please show the optimal decisions and total cost borne by the carrier. (10%)

   (e) If the margin of Plant 3 decreases to \$20, does the basis obtained in part (d) of the carrier's problem remain optimal? Explain. (10%)

Table 1: Information of resources, margins, and shipping costs

(a) Resource requirements and margins

| | Resources | | | |
|---|---|---|---|---|
| Plant | A | B | C | Margin |
| Plant 1 | 2 | 4 | 2 | \$40 |
| Plant 2 | 3 | 2 | 3 | \$30 |
| Plant 3 | 2 | 3 | 4 | \$40 |
| Plant 4 | 2 | 3 | 3 | \$20 |
| Available amount | 280 | 320 | 300 | |

(b) Shipping costs and path capacities

| | Plant 1 | Plant 2 | Plant 3 | Plant 4 | Market |
|---|---|---|---|---|---|
| Plant 1 | – | \$7 ($\infty$) | \$4 ($\infty$) | \$4 (20) | – |
| Plant 2 | – | – | \$3 (30) | \$3 (40) | – |
| Plant 3 | – | – | – | – | \$5 (50) |
| Plant 4 | – | – | \$2 ($\infty$) | – | \$5 ($\infty$) |

Note: – unavailable path; (·) path capacity

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2. Consider the inventory control problem of a product of a store. The inventory of the product is checked at the end of each day, if the inventory level is less than s, we place a replenishment order to bring the inventory level up to S. The replenishment will arrive at the store at the beginning of the next day. Let $D_n$ be random demand at day n with probability $P(D_n = k) = d_k, k = 0, 1, 2, \ldots$. $D_1, D_2, \ldots$ be iid random variables. Assume that unsatisfied demands are lost with a cost of \$b per unit of unsatisfied demand. Besides, there is a setup cost \$M of placing a replenishment order and a holding cost \$h per unit charged for each unit of inventory held at the end of the day. Let $X_n$ be inventory level of the product at the end of day n. $\{X_n\}$ is a Markov chain.

(a) Find the transition probability matrix of $\{X_n\}$.(% 15)

(b) Assume that $\{X_n\}$ has steady state distribution $\lim_{n\to\infty} P(X_n = i) = \pi_i$, express the following terms in terms of \$M, \$b, \$h, $\pi_i, i = 0, 1, 2, \ldots$

- Average inventory level. (% 5)
- Average unmet demand per day. (% 5)
- Average replenishment order size.(% 5)
- Average time between placing replenishment orders.(% 5)
- Average system cost per day.(% 10)

(c) If the product considered is perishable, say the product are boxes of milk. Assume each box of milk has to be sold within three days after it arrives to the store and we always sell boxes of milk in the shop longest of time first. If a box of milk has been in store for more than 3 days, we need to discard it. The status of each box of milk is checked at the end of each day. The inventory system of the product considered is still a Markov chain. Define the state of this Markov chain and find the transition probabilities. (% 5)