



本試題共 8 題，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分。

- 1 Solve the initial value problem: $\sin(x-y) + \cos(x-y) - \cos(x-y)y' = 0$; $y(0) = 7\pi/6$. (Hint: multiply the equation by an integrating factor to make the equation exact) (本題 10 分)
- 2 Find the general solution of the differential equation: $y'' - y = 2\sin^2(x)$. (DO NOT use the Laplace transform method) (本題 10 分)
- 3 Solve the initial value problem: $x^2y'' - 6y = 8x^2$; $y(1) = 1$, $y'(1) = 0$. (本題 10 分)
- 4 Use the Laplace transform to solve the equation: $f(t) = \cos(t) + e^{-2t} \int_0^t f(\alpha)e^{2\alpha} d\alpha$. (本題 10 分)
- 5 Find the inverse Laplace transform of $F(s) = \frac{e^{-2s}}{s^2(s+3)^2}$. (本題 10 分)
- 6 Let $f(t) = 1$, $0 \leq t \leq \pi$, find the Fourier cosine series and the Fourier sine series of $f(t)$ on interval $[0, \pi]$. (本題 15 分)
- 7 Let $A = \begin{bmatrix} 8 & -4 & 3 \\ 1 & 5 & -1 \\ -2 & 6 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $B = \begin{bmatrix} 0 \\ -5 \\ -4 \end{bmatrix}$: (本題 15 分)
 - (1) find the determinant ($|A|$) of the matrix A , and find the solution of $AX = B$ by Cramer's rule. (本小題 8 分)
 - (2) Find the inverse matrix (A^{-1}) of the matrix A , and find the solution of $AX = B$ by $X = A^{-1}B$. (本小題 7 分)
- 8 Let $A = \begin{bmatrix} 7 & -1 \\ 1 & 5 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, and $X(t=0) = X(0) = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$: (本題 20 分)
 - (1) find the eigenvalues and eigenvectors of A . (本小題 5 分)
 - (2) find a fundamental matrix ($\Omega(t)$) for the systems of linear differential equations, $X' = AX$. (本小題 5 分)
 - (3) find the general solution of the system $X' = AX$. (本小題 5 分)
 - (4) solve the initial value problem of $X' = AX$ with $X(0)$. (本小題 5 分)