## 國 立 雲 林 科 技 大 學 <br> 系所：資工系 <br> 101 學年度碩士班暨碩士在職專班招生考試試題 <br> 科目：離散數學

1．Show that any Hamiltonian cycle in the following graph that contains the edge $\alpha$ must also contain the edge $\beta$ ．（ $10 \%$ ）


2．Solve the recurrence relation and find the value of $a_{16}$ ，where $a_{n+1}^{2}=5 a_{n}^{2}, a_{n} \geq 0, a_{0}=3$ ．（ $15 \%$ ）

3．Determine the generating function for the sequence： $\mathbf{0}, \mathbf{0}, \mathbf{1}, \mathbf{0}, \mathbf{0}, \mathbf{1}, \mathbf{0}, \mathbf{0}, \mathbf{1}, \ldots(15 \%)$

4．（a）If the in－order and post－order results of a binary tree T are CBFDGA and CFGDBA， respectively，please determine the binary tree T，where $\{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}, \mathrm{G}\}$ are tree nodes．（5\％）
（b）Meanwhile，list the pre－order of the binary tree T．（5\％）

5．The population of Olympia is approximately 18，273．Show that at least two people in Olympia have the same initials．（Note that some people do not have middle names．）（5\％）

6．Define the relation R on Z to be $\mathrm{a} \mathrm{R} b$ if $\mathrm{a}-\mathrm{b}$ is prime．Is R reflexive？Symmetric？Transitive？ Explain why！（5\％）

7．Let $\Sigma=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}\}$ ．（a）What is $|\Sigma 2|$ ？$|\Sigma 3|$ ？（b）How many strings in $\Sigma^{*}$ have length at most 5 ？ （8\％）

8．Verify that the expression $(p \Rightarrow q) \Leftrightarrow(\neg p \vee q)$ is a tautology．（7\％）

9．Write a Turing machine that，when run on the tape（ $8 \%$ ）
．．．bl110b．．．
will produce an output tape of
．．．b11101b．．．

10．Consider the open statement（ $10 \%$ ）

$$
p(x, y): \quad y-x=y+x^{2}
$$

where the universe for each of the variables $x, y$ comprises all integers．Determine the True or

## H <br> 國立雲林科技 大 學 <br> 系所：資工系 <br> 101 學年度碩士班暨碩士在職專班招生考試試題 科目：離散數學

False value for each of the following statements：
A）$p(0,1)$
B）$\forall y p(0, y)$
C）$\exists y p(1, y)$
D）$\forall x \exists y p(x, y)$
E）$\exists y \forall x p(x, y)$

11．Please minimize the finite state machine shown below．（7\％）

|  | Next State |  | Output |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 0 | 1 |
| $S_{1}$ | $S_{4}$ | $S_{3}$ | 0 | 0 |
| $S_{2}$ | $S_{5}$ | $S_{2}$ | 1 | 0 |
| $S_{3}$ | $S_{2}$ | $S_{4}$ | 0 | 0 |
| $S_{4}$ | $S_{5}$ | $S_{3}$ | 0 | 0 |
| $S_{5}$ | $S_{2}$ | $S_{5}$ | 1 | 0 |
| $S_{6}$ | $S_{1}$ | $S_{6}$ | 1 | 0 |

