國立雲林科技大學 系所：資工系
101 學年度碩士班暨碩士在職專班招生考試試題 科目：線性代數(2)

本試題共九題，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分。

1. (20%) Let P_n denote the s et of real polynomial functions of degree $\leq n$.
   (a) (10%) Show that the set $\{x^2+1, 3x-1, -4x+1\}$ is linearl y independent in P_2.
   (b) (10%) Show that the set $\{x+1, x-1, -x+5\}$ is linearl y dependent in P_1.

2. (10%) Find the reduced echelon form for each of the following matrices. Use the echelon form to determine a basis for the row space, and the rank of each matrix.
   (a) (5%) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 3 & 4 & 7 \end{bmatrix}$
   (b) (5%) $\begin{bmatrix} 1 & 1 & 0 & -1 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 0 & -1 \\ -1 & 0 & 1 & 1 \end{bmatrix}$

3. (20%) Let $T: U \rightarrow V$ be a linear transformation. Let T be defined relative to bases $\{\mathbf{u}_1, \mathbf{u}_2\}$ and $\{\mathbf{v}_1, \mathbf{v}_2\}$ of U and V as follows:
$$T(\mathbf{u}_1) = 2\mathbf{v}_1 + 3\mathbf{v}_2, \quad T(\mathbf{u}_2) = 4\mathbf{v}_1 - \mathbf{v}_2.$$
   (a) (10%) Find the matrix of T with respect to these b ases.
   (b) (10%) Use this matrix to find the image of the vector $\mathbf{u} = 2\mathbf{u}_1 + 5\mathbf{u}_2$.

4. (8%) Please answer:
   (a) (4%) Determine the matrix of coefficients and augmented matrix of each following system of equation.
$$\begin{cases} x_1 + 2x_2 + 3x_3 = 14 \\ 2x_1 + 5x_2 + 8x_3 = 36 \\ x_1 - x_2 = -4 \end{cases}$$
   (b) (4%) Solve the system using the method of Gauss-Jordan elimination with matrices.

5. (6%) Find the image of the triangle having the following vertices A(1, 2), B(2, 8), C(3, 2) under the rotation of $\pi/2$ with respective to point P(5, 4).

6. (12%) Evaluate the determinants of the following matrices.
   (a) $\begin{bmatrix} 0 & 3 & 2 \\ 1 & 5 & 7 \\ -2 & -6 & -1 \end{bmatrix}$

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(b) $\begin{bmatrix} 1 & -2 & 3 & 0 \\ 4 & 0 & 5 & 0 \\ 7 & -3 & 8 & 4 \\ -3 & 0 & 4 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 5 & 1 & 0 \\ 0 & 3 & 2 & -7 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -2 & 3 \\ 7 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

7. (10%) Consider the matrix $A = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$.

   (a) (5%) Find its eigenvalues.

   (b) (5%) Find the corresponding normalized eigenvectors.

8. (8%) If $A^{-1} = \begin{bmatrix} 3 & 4 \\ -1 & -1 \end{bmatrix}$; find A.

9. (6%) Determine whether the following matrices are singular. Give the reason.

   (a) $\begin{bmatrix} 1 & 5 & 5 \\ 0 & -2 & -2 \\ 3 & 1 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 7 & 9 & 0 \\ -2 & 3 & 0 \\ 4 & 5 & 0 \end{bmatrix}$