



There are totally 7 questions, totally 100 points. Please answer the following questions in order, otherwise no score will be considered.

- Determine the fundamental period of the signals below:
  - (3%) Determine the fundamental period of the signal  $2 \sin(3t + 2)$ ?
  - (3%) Determine the fundamental period of the signal  $3 \cos(4t + 3)$ ?
  - (4%) Determine the fundamental period of the signal  $[2 \sin(3t + 2) - 3 \cos(4t + 3)]$ ?
- Consider the feedback system as Figure 1. Assume that  $y[n] = 0$  for  $n < 0$ :
  - (5%) Sketch the function of the output  $y[n]$  when  $x[n] = \delta[n]$ ?
  - (5%) Sketch the function of the output  $y[n]$  when  $x[n] = u[n]$ ?

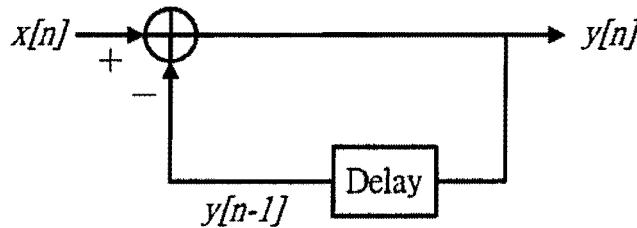


Figure 1

- Let  $x(t)$  be the rectangular pulse shown in Figure 2(a), and let  $h(t)$  be the impulse train depicted in Figure 2(b). That is

$$h(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

Determine and sketch  $y(t) = x(t) * h(t)$  when  $T$  is equal to the following value:

- (5%)  $T = 4$ ?
- (5%)  $T = 2$ ?
- (5%)  $T = 3/2$ ?
- (5%)  $T = 1$ ?

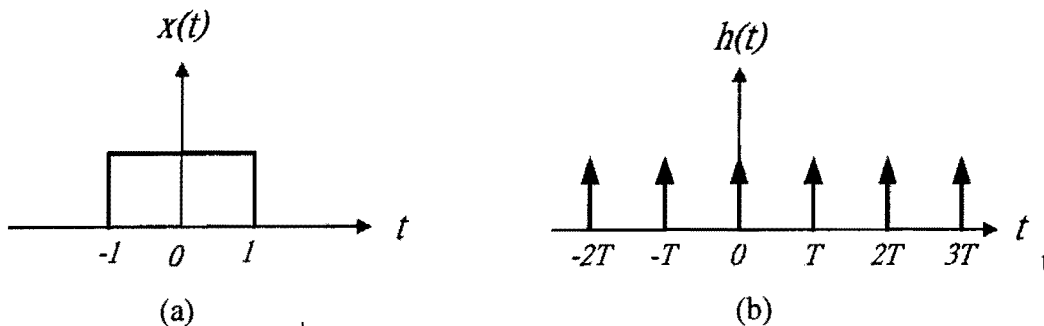


Figure 2

- Let  $x(t)$  be a periodic signal with fundamental period  $T$  and Fourier series coefficients  $a_k$ . That is,  $a_k = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt$  and  $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t}$ . Please derive the Fourier series coefficients of the following signals in terms of  $a_k$ :

- (5%)  $x(5t - t_0)$ ?
- (5%)  $\frac{d^2 x(t)}{dt^2}$ ?



5. (16%) AM Modulation. Given a voice signal  $s(t)$  whose Fourier Transform (spectrum)  $S(\omega)=10$  between the range  $(-\omega_1, \omega_1)$  and  $S(\omega)=0$  otherwise. Let  $p(t)=\cos \omega_0 t$  be the modulation signal, assume  $\omega_0 \gg \omega_m$

(a) Sketch  $S(\omega)$ .

(b) Sketch  $P(\omega)$ , the Fourier Transform of  $p(t)$  (Hint:  $P(\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$ )

(c) The transmitted signal  $r(t)=s(t)p(t)$ , find and sketch  $R(\omega)$ , the Fourier Transform of  $r(t)$ .

(d) The demodulated signal  $g(t)=r(t)p(t)$ , find and sketch  $G(\omega)$ , the Fourier Transform of  $g(t)$ .

How to recover  $s(t)$  from  $g(t)$ ?

(Hint: Multiplication in time domain corresponds to convolution in the frequency domain)

6. (16%) Digitization is the process to obtain a discrete time signal  $x[n]$  from a continuous time signal  $x(t)$ .

(a) Name the two major steps in digitization.

(b) Why do digital processing of continuous time signals become standard in most applications?

(c) Given a signal  $x(t)$  with non-zero frequency contents between  $(-2\pi \cdot 4000, 2\pi \cdot 4000)$ .

Accordingly to the sampling theorem, what is the minimal sampling period  $T$  so that  $x(t)$  can be reconstructed from  $x[n]$  without aliasing?

7. (18%) Aliasing as Figure 3

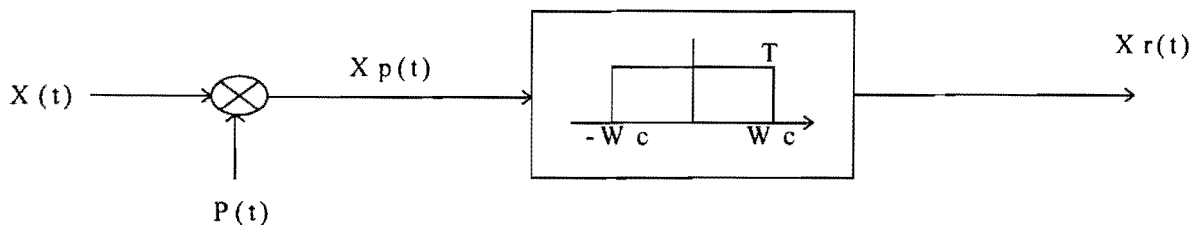


Figure 3

where:

$$\begin{cases} X(t) = \cos \omega_0 t, & \text{Sampling function } P(t) = \sum_{k=-\infty}^{k=+\infty} \delta(t - kT) \\ Xp(t) = X(t)P(t), & \omega_c = \frac{\omega_s}{2}, \quad \omega_s = \frac{2\pi}{T} = 4000 \end{cases}$$

Find  $X_r(t)$  for each  $\omega_0$  given below (explain the reasons, no score if guessing)

(a)  $\omega_0=100$  (b)  $\omega_0=150$  (c)  $\omega_0=300$  (d)  $\omega_0=400$  (e)  $\omega_0=500$