國立雲林科技大學
101 學年度碩士班暨碩士在職專班招生考試試題

系所：電機系
科目：線性代數(1)

(1) Determine the **values** of k such that the following equations $\begin{cases} 4x+ky=6 \\ kx+y=-3 \end{cases}$

(i) the equation has **no solution.** (4%)

(ii) the equation has **exact one solution.** (4%)

(iii) the equation has **an infinite number of solutions.** (4%)

(2) Determine the polynomial $p(x)=a_0+a_1x+a_2x^2$ whose graph passes the given points (1, 2), (2, 0), (3, 4). (12%)

(3) Solve the following linear system $\mathbf{Ax}=\mathbf{b}$ with LU-Factorization of

$$\begin{aligned} 2x_1+x_2 \quad &= 1 \\ x_2-x_3 &= 2 \\ -2x_1+x_2+x_3 &= -6 \end{aligned}$$

(i) Find the LU-Factorization of the coefficient matrix $\mathbf{A}$, where diagonal elements of L are 1. (8%)

(ii) From (i), solving $\mathbf{y}$ of the lower triangular system $L\mathbf{y}=\mathbf{b}$, where $\mathbf{y}=U\mathbf{x}$. (4%)

(iii) From (i) and (ii), solving $\mathbf{x}$ of the upper triangular system $U\mathbf{x}=\mathbf{y}$. (4%)

(4) Express the vector $\mathbf{b}$ as a linear combination of the columns of $\mathbf{A}$.

$$\mathbf{A}=\begin{bmatrix} 1 & 1 & -5 \\ 1 & 0 & -1 \\ 2 & -1 & -1 \end{bmatrix}, \quad \mathbf{b}=\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}.$$ (10%)

(5) Let $\boldsymbol{T}$ be the triangle with vertices at $(x_1, y_1), (x_2, y_2), (x_3, y_3)$. Show that

$$\{\text{area of } \boldsymbol{T}\}=\frac{1}{2}\left|\det\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}\right|$$ (10%)

(6) Find the area of the region $\boldsymbol{E}$ bounded by the ellipse whose equation is

$$\frac{x^2}{4}+\frac{y^2}{9}=1.$$ (10%)

(7) (a) Find rank **A** and dim Null **A**. (8%)

(b) Find bases for the row space, the column space, and the null space of the matrix **A**.

$$\mathbf{A} = \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix}$$ (12%)

(8) Compute $\mathbf{A}^{10}$ where $\mathbf{A} = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$. (Hint: utilize $\mathbf{A} = \mathbf{PDP}^{-1}$) (10%)