



Prob. 1 (30%)

The model of a motor with input voltage  $f(t)$  and output angular velocity  $y(t)$  is given as following,  $y' + y = f(t)$ , with  $y(0) = 0$  where  $f(t)$  is given in figure 1.

(1) Please solve  $y(t)$  by Laplace transform. (15%)

(2) Please solve  $y(t)$  by convolution integral (15%)

defined as  $\int_0^t g(t-\tau)f(\tau)d\tau$ .

Where  $g(t)$  is the unit impulse response, that is the solution of  $y' + y = \delta(t)$

with the unit impulse function (Dirac delta function)  $\delta(t)$ .

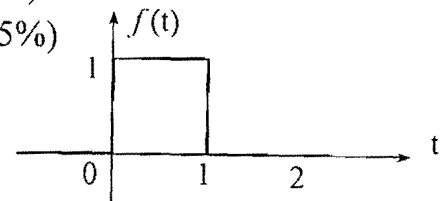


Figure 1

Prob. 2 (20%)

Given an equations as below

$$y' + 2y = 2 \cos(2x)\sin(4x),$$

please solve its particular solution  $y_p(x)$ .



3. Consider the system of linear equations:

$$2X_1 + X_2 - 11X_3 = -6$$

$$-5X_1 + X_2 + 9X_3 = 12$$

$$X_1 + X_2 + 14X_3 = -5$$

The system can be written in matrix form as  $AX = B$ . Please find (1) A, B, (2) the Rank of matrix A by Gauss elimination, (3)  $\det(A)$ , (4)  $A^{-1}$ , (5)  $A^{-1}B$ , (6) X. (25%)

4. Solve  $\frac{\partial u}{\partial t} = 9 \frac{\partial^2 u}{\partial x^2}$  for  $0 < x < 3, t > 0$

with  $u(0,t) = u(3,t) = 0$  for  $t \geq 0$ ;  $u(x,0) = 50$  for  $0 \leq x \leq 3$  (25%)