

國立高雄應用科技大學
101 學年度碩士班招生考試
機械與精密工程研究所

准考證號碼 (考生必須填寫)

工程數學 (甲、乙組)

試題 共 2 頁，第 1 頁

注意：a. 本試題共 8 題，共 100 分。

b. 作答時不必抄題。

c. 考生作答前請詳閱答案卷之考生注意事項。

1. Solve the following first-order differential equation. (10%)

$$3y^4 - 1 + 12xy^3 y' = 0; \quad y(1) = 2$$

2. Find the general solution of the following differential equation. (10%)

$$y'' + \frac{1}{x} y' - 16 \frac{1}{x^2} y = 0$$

3. (a) Let ℓ denote the operator of the Laplace transform, show that the Laplace transform of f' is

$$\ell\{f'\} = s\ell\{f\} - f(0) \quad (5\%)$$

(b) Solve the following initial value problem by Laplace transform method. (15%)

$$y' + 0.2y = 0.01t, \quad y(0) = -0.25$$

4. Find the straight line L_1 through the point P: (1, 3) in the xy-plane and perpendicular to the straight line $L_2: x - 2y + 2 = 0$. (10%)

5. Evaluate the line integral $\int_C (x + y) ds$ over the curve C, where C is given by $x =$

$$y = t, \quad z = t^2 \quad \text{for } 0 \leq t \leq 2. \quad (10\%)$$

6. (a) Find the eigenvalues and its corresponding eigenvectors of the following matrix A. (10%)

$$A = \begin{pmatrix} 3 & 3 \\ 1 & 5 \end{pmatrix}$$

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(b) Solve the general solution for the following linear system of equation with the matrix A in (a). (10%)

$$X' = AX$$

Where $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

7. Evaluate the line integral $\oint_C \vec{F}(r) \cdot d\vec{r}$ counterclockwise around the boundary C

of the region R , where

$$\vec{F}(r) = y\vec{i} - x\vec{j}, C \text{ the circle } x^2 + y^2 = \frac{1}{4}. \text{ (10\%)}$$

8. Let $z_1 = -2 + 2i$ and $z_2 = 3i$.

(a) Find $\frac{z_1}{z_2}$. (5%)

(b) Find $\sqrt[3]{\frac{z_1}{z_2}}$. (5%)