國立新竹教育大學 101 學年度碩、博士班招生考試試題

所別：應用數學系碩士班

科目：微積分(本科總分 150 分，含初等微積分、高等微積分)

※ 請橫書作答

1. Find the following limit

(a) $\lim_{h\to 0}\frac{\sqrt{1+h}-1}{h}$ (b) $\lim_{x\to\infty}\left(e^{x}+x\right)^{1/x}$ (12 points)

2. In the following, find $\frac{dy}{dx}$ (18 points)

(a) $y\sin x^2 = x\sin y^2$ (b) $y=(\sin x)^{\ln x}$ (c) $y=\int_{\tan x}^{x^2}\frac{1}{\sqrt{2+t^4}}dt$

3. Evaluate the following integral (20 points)

(a) $\int_{e}^{e^4}\frac{1}{x\sqrt{\ln x}}dx$ (b) $\int e^{x}\sqrt{1+e^{x}}dx$ (c) $\int\frac{\sin^3 x}{\cos x}dx$ (d) $\int_0^3\frac{1}{x^2-6x+5}dx$

4. The region D enclosed by the curves $y=x$ and $y=x^2$ is rotated about the line $y=2$. Find the volume of the resulting solid. (10 points)

5. Determine whether the series is convergent or divergent. (15 points)

(a) $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$ (b) $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}$ (C) $\sum_{n=1}^{\infty}(-1)^n\sin\left(\frac{\pi}{n}\right)$

6. Suppose A, B are open sets in $\mathbb{R}$ and F, G are closed sets in $\mathbb{R}$.

   (a) Prove that $A\cup B$ and $A\cap B$ are open. (10 points)

   (b) Prove that $F\cap G$ is closed. (5 points)

7. Suppose $\{x_n\}$ is a sequence of real numbers such that

(a) $|x_n - x_{n+1}| \le \frac{1}{n+5}$ prove or disprove that $\{x_n\}$ converges. (10 points)

(b) $|x_n - x_{n+1}| \le \frac{1}{n^2}$ prove or disprove that $\{x_n\}$ converges. (10 points)

8. (a) Find an open cover of $[0,1)$ with no subcover in $\mathbb{R}$. (10 points)

(b) Is $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n}\cdots\right\}$ compact in $\mathbb{R}$? Justify your answer. (10 points)

9. (a) Let $\{A_k\}$ be a sequence of compact nonempty sets in $\mathbb{R}$ such that $A_{k+1} \subset A_k$ for all $k \in \mathbb{N}$. Prove that $\bigcap_{k\in\mathbb{N}} A_k \ne \phi$. (10 points)

(b) Find an example of $\{A_k\}$ in $\mathbb{R}$ such that $A_{k+1} \subset A_k$ for all $k \in \mathbb{N}$ but $\bigcap_{k\in\mathbb{N}} A_k = \phi$. (10 points)