國立彰化師範大學 101 學年度<u>碩士班</u>招生考試試題

系所：　<u>電子工程學系</u>　　　　組別：　<u>甲、乙組</u>　　　　科目：　<u>工程數學</u>

1. Solve the following equations.

(1) $x^2 y'' - 6y = 0$ (5%)

(2) $y'' - 4y = \cos x$ (5%)

(3) $y'' - 2y' - 8y = 0$ (5%)

(4) $y''' - 7y'' - 11y' - 6y = e^{2x}$ (5%)

(5) $e^y (\cos x\, dx + \sin x\, dy) = 0$ (5%)

(6) $e^{-y} dx + e^{-x} (e^{-y} + 2) dy = 0$ (5%)

2. Solve the following problems by Laplace transform.

(1) $y'' + 9y = 2e^{-t}$, $y(0) = 0, y'(0) = 0$ (10%)

(2) $y'' + 4y = r(t)$, $r(t) = \begin{cases} 3\sin t, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$ and $y(0) = 2, y'(0) = 3$ (10%)

3. (a) Show that the Fourier series for the periodic function $f(x) = \begin{cases} -x, & -2 \le x < 0 \\ x, & 0 \le x < 2 \end{cases}$ with

$f(x+4) = f(x)$ for all x. (b) From the Fourier series of (a), please deduce the value of the sum of

the series $1 + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \dfrac{1}{7^2} + \cdots = \displaystyle\sum_{n=0}^{\infty} \dfrac{1}{(2n+1)^2}$. (20%)

4. Suppose $\vec{F} = x\hat{i} + y\hat{j} + (z^2 + 1)\hat{k}$ and S is the surface of the region bounded by $x^2 + y^2 = a^2$,

$z = 0$, $z = c$. Evaluate $\iint_S \left(\vec{F} \bullet \hat{n} \right) dS$ where $\hat{n}$ is a unit normal to S. (15%)

5. Evaluate the Cauchy principle value of the improper integral

$\displaystyle\int_{-\infty}^{\infty} \dfrac{a\cos x + x\sin x}{x^2 + a^2} dx, \ a > 0.$ (15%)