系所： <u>車輛科技研究所</u>　　　　　　　　　　　　　　科目： <u>自動控制</u>

☆☆請在答案卷上作答☆☆　　　　　　　　　　　　　　共 **1** 頁，第 **1** 頁

1. Given the differential equation $\dfrac{d^2 y}{dt^2} + 12\dfrac{dy}{dt} + 32y = 32u(t)$, solve for $y(t)$ using the Laplace transform, if all initial conditions are zero. **(20%)**

2. Find the eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}$. **(20%)**

3. Find the transfer function $T(s) = Y(s)/U(s)$, where $U(s)$ is the input and $Y(s)$ is the output, given the system defined by $\dot{\mathbf{X}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} x + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u$, $y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$. **(20%)**

4. A unity feedback system has the forward transfer function $G(s) = \dfrac{10(s+20)(s+30)}{s(s+25)(s+35)}$, find the steady-state error for the following inputs $15u(t),\ 15t\,u(t),\ 15t^2\,u(t)$. **(20%)**

5. Given the system represented in state space $\dot{x} = \begin{bmatrix} 0 & 2 \\ -2 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-2t}$, $y = \begin{bmatrix} 2 & 1 \end{bmatrix} x$, $x(0)=\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

   do the following (a) solve for the state-transition matrix. **(6%)**
   　　　　　　　(b) solve for the state vector using the convolution integral. **(7%)**
   　　　　　　　(c) find the output $y(t)$. **(7%)**