1. Consider the matrix $A = \begin{bmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{bmatrix}$, find the *eigenvalues* and the associated *eigenvectors* of matrix A. (10%)

2. Solve the linear system $\begin{cases} x + 2y - 3z + 5w &= 0 \\ 2x + y - 4z - w &= 1 \\ x + y + z + w &= 0 \\ -x - y - z + w &= 4 \end{cases}$. (10%)

3. (a) Find a vector *orthogonal* to the vectors **u**=[2, 1, -1] and **v**=[1, 2, 1]. (5%)

   (b) Let $A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 1 \\ 2 & -1 & 1 \end{bmatrix}$ be the coefficient matrix of a *homogeneous system* in **x**=[x_1, x_2, x_3], show that the system *Ax=0* has a unique solution. (5%)

4. Define the ***absolute determinant*** of a *mXn* matrix *A* ($n \leq m$) (denoted as ***abs_det(A)***) is given by the definition ***abs_det(A)=det(A^TA)$^{1/2}$***.

   For example $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 2 \end{bmatrix}$, ***abs_det(A)***$= (det(A^TA))^{1/2} = \left(\det\left(\begin{bmatrix} 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right) \right)^{1/2} = \sqrt{\det([5])} = \sqrt{5}$.

   Let $B = \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ 1 & 4 \end{bmatrix}$, find ***abs_det(B)***. (10%)

5. Use the *Gram-Schmidt process* to transform [1, 0, 1], [1, 2, -2], [2, -1, 1] into an orthogonal basis for R^3. (10%)

6. Let $X = \{1, 2, 3, \ldots, 10\}$. Define a relation R on $X \times X$ by $(a, b)R(c, d)$ if $a+d=b+c$. Please show that R is an equivalence relation on $X \times X$. (10%)

7. Show that if a, b, k, and m are integers such that $k \geq 1$, $m \geq 2$, and $a \equiv b \pmod{m}$, then $a^k \equiv b^k \pmod{m}$ whenever k is a positive integer. (10%)

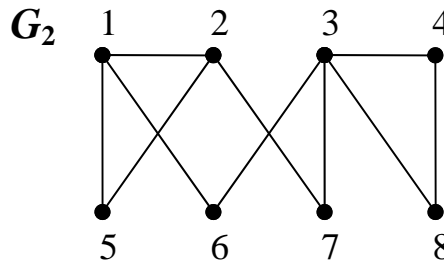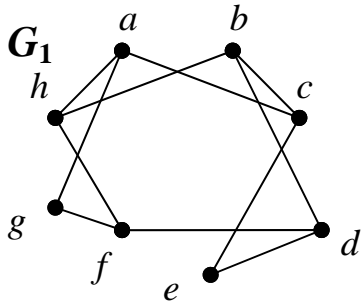**[考生注意!!第二面尚有試題]**

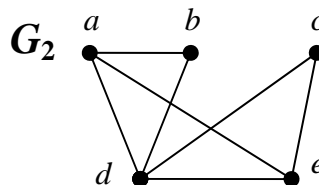系所： 資訊工程學系　　　　　　　　　　　科目： 離散數學及線性代數
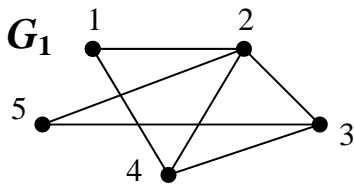
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8. Determine whether the graphs G_1 and G_2 are isomorphic. If the graphs are isomorphic, explain your reason; otherwise, give an invariant that the graphs do not share. (10%)

(a)

(b)

9. Find the coefficient of the specified term when the expression is expanded.
(a) $w^2x^3y^2z^5$; $(2w+x+3y+z)^{12}$. (5%)
(b) a^2x^3; $(a+x+c)^2(a+x+d)^3$. (5%)

10. Prove that if n is an integer greater than 1, then n can be written as the product of primes. (10%)