國立彰化師範大學 101 學年度碩士班招生考試試題

系所： 物理學系　　　　組別： 甲組　　　　科目： 物理數學

☆☆請在答案卷上作答☆☆　　　　

1. (20%) Solve the given differential equation :

   (a) $y'' - 2y' - 3y = 0,\ y(0) = 2,\ y'(0) = 14.$　　　(b) $x^2 y' + 3xy = \dfrac{1}{x},\ y(1) = -1.$

2. (15%) Using the convolution theorem, solve: $y(t) + 2e^t \int_0^t e^{-\tau} y(\tau) d\tau = te^t.$

3. (15%) Find the Fourier series of $f(x) = x^2$, if $-1 < x < 1$, and $f(x+2) = f(x)$, period $p = 2\pi$.

4. (15%) A complex function $f(z)$ is defined as $f(z) = \dfrac{7z-3}{z(z-1)}$. Expand $f(z)$ in a Laurent series valid for (a) $0 < |z| < 1$ and (b) $0 < |z-1| < 1$. (c) Also, evaluate the integral $\oint_C f(z)dz$, where C is the counterclockwise oriented circle $|z| = 2$.

5. (15%) Consider a 1-D wave equation with a forcing term and homogeneous initial and boundary conditions:

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2} - \sin(5x), \text{ for } 0 < x < \pi,\ t > 0,$$

$$u(x,0) = \frac{\partial u(x,0)}{\partial t} = 0, \text{ for } 0 \le x \le \pi, \text{ and } u(0,t) = u(\pi,t) = 0, \text{ for } t > 0.$$

   (a) By applying an ansatz of the form $u(x,t) = v(x,t) + w(x)$ with a suitably chosen function $w(x)$, transform the wave equation into a problem for $v(x,t)$, where $v(x,t)$ satisfies a homogeneous wave equation with inhomogeneous initial conditions.
   (b) Solve the problem for $v(x,t)$ by separation of variables.
   (c) Show explicitly that the expression for $u(x,t)$ obtained in this way satisfies all the conditions mentioned in this problem.

6. (20%)
   (a) Express the real quadratic form $F = 13x_1^2 + 13x_2^2 - 10x_1x_2$ in terms of a real symmetric matrix $\mathbf{A}$ and column vector $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ as $F = \mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x}$.
   (b) Find the eigenvalues and normalized eigenvectors of $\mathbf{A}$.
   (c) By performing an orthogonal transformation to a new vector $\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$, $\mathbf{x} = \mathbf{R}\mathbf{y}$ with $\mathbf{R}^{\mathrm{T}}\mathbf{R} = \mathbf{I}$, F can be written in the diagonal form $F = ay_1^2 + by_2^2$. If $a < b$, what are the constants a and b?
   (d) Express y_1 and y_2 in terms of x_1 and x_2.