國立彰化師範大學 101 學年度碩士班招生考試試題

系所：數學系　　組別：甲、乙、丙組　　科目：線性代數

☆☆請在答案卷上作答☆☆　　共 1 頁，第 1 頁

1. Find a basis for the subspace

$$\left\{ \begin{pmatrix} a-2b+5c \\ 2a+5b-8c \\ -a-4b+7c \\ 3a+b+c \end{pmatrix} : a,b,c \in \mathbb{R} \right\}. \quad (10\%)$$

2. Let $A=(a_{ij}) \in M_{m\times n}(F)$. Show that the row space $R(A)=\text{span}(\{(a_{11},a_{12},\cdots,a_{1n}),$ $(a_{21},a_{22},\cdots,a_{2n}),\ \cdots,\ (a_{m1},a_{m2},\cdots,a_{mn})\})$ of *A* is orthogonal to the null space $N(A)$ of *A*. (10%)

3. A linear transformation $T: P_2(\mathbb{R}) \to P_3(\mathbb{R})$ is defined as $T(f(x))=(x^2 f(x))'$. Find

   (a) its matrix representation for the standard ordered bases $\beta_1=\{1,x,x^2\}$ of $P_2(\mathbb{R})$ and $\beta_2=\{1,x,x^2,x^3\}$ of $P_3(\mathbb{R})$. (10%)

   (b) a basis for $\ker(T)$. (10%)

   (c) a basis for range of *T*. (10%)

4. Let *V* be an inner product space, and let *T* be a linear operator on *V*. Show that if *T* is a normal projection, then $R(T)=N^{\perp}(T)$. (15%)

5. Let $P_n(R)$ denote the vector space of polynomials of degree at most n and with coefficients in the set R of real numbers. For a fixed $a \in R$, determine the dimension of the subspace of $P_n(R)$ defined by $\{f \in P_n(R): f(a)=0\}$. (10%)

6. Let $A=\begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix}$. Compute A^n for any positive integer *n*. (10%)

7. Prove the determinant evaluation: $\begin{vmatrix} 1 & c_0 & c_0^2 & \cdots & c_0^n \\ 1 & c_1 & c_1^2 & \cdots & c_1^n \\ 1 & c_2 & c_2^2 & \cdots & c_2^n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & c_n & c_n^2 & \cdots & c_n^n \end{vmatrix} = \prod_{0\le i<j\le n}(c_j-c_i)$. (15%)