

# 國立彰化師範大學 101 學年度碩士班招生考試試題

系所：數學系

組別：乙組

科目：高等微積分

☆☆請在答案卷上作答☆☆

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1. Use the  $\varepsilon - \delta$  definition of limit to show  $\lim_{x \rightarrow 0} \frac{1}{\sqrt{3x+5}} = \frac{1}{\sqrt{5}}$ . (15%)
2. (a) At what value  $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$  the series  $\sum_{n=1}^{\infty} \frac{\tan^n x}{\sqrt{n}}$  converges or diverges? Prove your answers. (15%)  
(b) Show the series  $\sum_{n=1}^{\infty} \frac{\tan^n x}{\sqrt{n}}$  converges uniformly on  $[-a, a]$  ( $0 < a < \frac{\pi}{4}$ ), and does it converge uniformly on  $[-\frac{\pi}{4}, a]$ ? (15%)
3. Let  $f$  be continuous real function on  $\mathbb{R}$ . Suppose that  $f'(x)$  exists for all  $x \neq 0$  and that  $f'(x) \rightarrow 3$  as  $x \rightarrow 0$ . Does  $f'(0)$  exist? Prove your answer. (20%)
4. Let  $f(t)$  and  $g(t)$  be real twice differentiable functions on  $\mathbb{R}$ . Put  $u(x, y) = f(x+ay) + g(x-ay)$ . Prove  $a^2 u_{xx} = u_{yy}$ . (15%)
5. Evaluate  $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+xy+y^2)} dx dy$ . (20%)  
(Hint: use change of variables formula for functions of two variables, and  $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$ )