國立彰化師範大學 101 學年度碩士班招生考試試題

系所：科學教育研究所　　組別：甲組　　科目：普通數學(含微積分及線性代數)

☆☆請在答案卷上作答☆☆　　　　　　　　　　　　　　　　　　　　　　　　共 1 頁，第 1 頁

1. (a) (10 points) State the *Mean-Value Theorem* for a real value function f defined on all real numbers.

   (b) (10 points) Show that if f is differentiable on (a,b) and $f'(x) \neq 0$ for all $x \in (a,b)$, then the equation $f(x) = 0$ has at most one real solution in (a,b).

2. (15 points) Find the enclosed area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

3. (a) (5 points) Taylor expand $\sin x$ at $x = 0$.

   (b) (5 points) Show that the Taylor series you derived is convergent.

   (c) (5 points) Use this Taylor series to show that $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$.

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4. (10 points) Solve the system of linear equations:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 & = & 1 \\ 2x_1 + x_2 + x_3 + x_4 & = & -2 \\ x_1 + 2x_2 + x_3 - x_4 & = & 3 \end{cases}$$

5. Consider the polynomial space of degree 2: $P_2(\mathbf{R}) = \{a + bx + cx^2 : a, b, c \in \mathbf{R}\}$ where $\mathbf{R}$ denote the set of real numbers.

   (a) (10 points) Find a basis $\beta = \{f_1, f_2, f_3\}$ for $P_2(\mathbf{R})$ such that

$$f_i(j) \quad = \quad \begin{cases} 0 & , \ \text{if } i \neq j \\ 1 & , \ \text{if } i = j \end{cases} \quad ,$$

   for $i, j \in \{1, 2, 3\}$.

   (b) (5 points) Find the coordinate vector of $p(x) = 2x^2 - x + 3$ corresponding to β, i.e. $[p(x)]_\beta$.

6. (15 points) Let

$$A = \begin{pmatrix} 0 & 3 & 1 \\ 2 & 1 & 1 \\ -2 & 3 & 3 \end{pmatrix}.$$

   Find a matrix Q such that $Q^{-1}AQ$ is a diagonal matrix.

7. (10 points) Find an orthonormal basis β for $P_2(\mathbf{R})$ with the inner product

$$< f, g >= \int_0^1 f(t)g(t)dt.$$