國立高雄師範大學 101 學年度碩士班招生考試試題

系所別：物理學系

科　目：物理數學（全一頁）

※注意：1. 作答時請將試題題號及答案依序寫在答案卷上，於本試題上作答者，不予計分。
2. 請以藍、黑色鋼筆或原子筆作答，以鉛筆或其他顏色作答之部份，該題不予計分。

1. Use the complex integral to evaluate $\int_{-\infty}^{\infty} \frac{1}{x-2i} dx$.（10%）
2. Use Green's theorem to evaluate $\oint_C x^2 y dx - xy^2 dy$, C is the boundary of the region $x^2 + y^2 \le 4, x \ge 0, y \ge 0$.（10%）
3. Solve the following O.D.E. $(x+2)^2 y'' - (x+1)y' + y = 0$.（10%）
4. Solve the following problem. $x^2 y'' + y' + \lambda y = 0, y(0) = y(1) = 0$（10%）
5. Find the eigenvalues and the orthogonal eigenvectors of the matrix.（10%）

$$A = \begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

6. The following are integral form of Maxwell's equations in vacuum. Use divergence theorem or Stokes' theorem to derive the differential form of Maxwell's equations in vacuum.（25%）

$$\oiint_S \vec{E} \cdot \hat{n} dA = \frac{1}{\varepsilon_0} \iiint_V \rho dV$$

$$\oiint_S \vec{B} \cdot \hat{n} dA = 0$$

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \iint_S \vec{B} \cdot \hat{n} dA$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 [\iint_S \vec{J} \cdot \hat{n} dA + \varepsilon_0 \frac{d}{dt} \iint_S \vec{E} \cdot \hat{n} dA]$$

The divergence theorem of a vector function $\vec{F}$: $\oiint_S \vec{F} \cdot \hat{n} dA = \iiint_V \nabla \cdot \vec{F} dV$

The Stokes' theorem of a vector function $\vec{F}$: $\oint_C \vec{F} \cdot d\vec{\ell} = \iint_S \nabla \times \vec{F} \cdot \hat{n} dA$

7. Find the eigenvalues and the corresponding normalized eigenvectors of the following matrix A and find the unitary transformation matrix U which makes $B = U^{-1} A U$ diagonal.（25%）

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & -ia \\ 0 & ia & 0 \end{pmatrix}$$ where a is a real constant.