國立高雄師範大學 101 學年度碩士班招生考試試題

系所別：數學系

科　目：線性代數

※注意：1. 作答時請將試題題號及答案依序寫在答案卷上，於本試題上作答者，不予計分。
2. 請以藍、黑色鋼筆或原子筆作答，以鉛筆或其他顏色作答之部份，該題不予計分。

1. Let V be a vector space over the scalar field F, and let $\{W_\lambda\}_{\lambda\in\Lambda}$ be any collection of subspaces of V. Prove that $W=\bigcap_{\lambda\in\Lambda}W_\lambda$ is a subspace of V.（15%）

2. Let V be a vector space over the scalar field F, and let S be a subset of V. The **subspace spanned** by S is defined to be the intersection of all subspaces of V which contain S. In this case, we also write span(S) to denote the subspace spanned by S. Prove that the subspace span(S) consists of all linear combinations of vectors in S.（15%）

3. Let W be a subspace consisting of all $n\times n$ symmetric matrices over the scalar field F.（10%）
   (i) Find the basis for W.
   (ii) Find the dimension of W.

4. Let V be a vector space over the scalar field F, and let U and W be two subspaces of V. We say that V is a direct sum of U and W if and only if, for every element $x\in V$, there exist unique elements $u\in U$ and $w\in W$ such that $x=u+w$. In this case, we write $V=U\oplus W$. Now, we assume that V is a finite-dimensional vector space over the scalar field F, and let U and W be two subspaces of V. Suppose that $V=U\oplus W$. Prove $\dim(V)=\dim(U)+\dim(W)$.（10%）

5. Let $S=\{(x,y,z)\in\mathbb{R}^3 : 5x-y=3z\}$.
   (a) Show that S is a vector space.（4%）
   (b) Find a basis for S.（4%）

6. Let V and W be vector spaces over a field F with zero vectors θ_V and θ_W, respectively. If $\dim(V)<\infty$ and $T:V\to W$ is a linear transformation, show that $\dim(V)=nullity(T)+rank(T)$.（15%）

（背面有題）

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7. Let $f:\ \mathbb{R}^{3}\to\ \mathbb{R}$ be a quadratic form defined by

$$f(x,y,z)=11x^2+5y^2+2z^2+16xy-20yz+4zx.$$

Find the maximum and minimum values of f subject to the constraint $x^2+y^2+z^2=1$, and determine the values of x, y and z at which the maximum and minimum occur.（15％）

8. Let W be a subspace of $\mathbb{R}^{n}$. For any vector $v\in\ \mathbb{R}^{n}$, show that the projection $Proj_W v$ of v on W is the unique vector in W that is closest to v.（12％）