國立臺灣師範大學 101 學年度碩士班招生考試試題

科目：工程數學（能源應用組）　　　　　適用系所：工業教育學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. Solve $\dfrac{dy}{dx} = (x+y)^2$　(10 分)

2. Solve $(x^2 + y^2)dy + xy \cdot dx = 0$　(10 分)

3. Solve the general solutions of: (共 15 分)

(a) $y'' - 7y' + 10y = 0$　(7 分)

(b) $(D+1)^2 (D-2)^2 (D^2 + 2D + 5)y = 0$, where $D \equiv \dfrac{d}{dx}$　(8 分)

4. Solve the general solution of : $y'' - y' - 6y = e^{3x}$　(10 分)

5. Prove the Laplace transform as follows: (共 10 分)

(a) $L[f(at)] = \dfrac{1}{a} F\left(\dfrac{s}{a}\right)$, where a is a constant. (5 分)

(b) $L\left[\int_0^t f(u)\,du\right] = \dfrac{1}{s} F(s)$　(5 分)

(Hint: $L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$)

6. Derive the inverse Laplace transform: $L^{-1}\left\{\dfrac{8s+1}{s^2 + s - 2}\right\}$　(10 分)

7. If a matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, please calculate A^{20}　(10 分)

8. Evaluate the inverse of: (a) $A_1 = \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix}$ (5 分) and (b) $A_2 = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 4 \\ 1 & 3 & 1 \end{bmatrix}$ (10 分)

(共 15 分)

9. If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$, and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$:

(a) Please prove that $z_1 z_2 = r(\cos\theta + i\sin\theta)$, where $r = r_1 \cdot r_2$, $\theta = \theta_1 + \theta_2$ (5 分)

(b) Please prove that $z_1/z_2 = r(\cos\theta + i\sin\theta)$, where $r = r_1/r_2$, $\theta = \theta_1 - \theta_2$ (5 分)

(共 10 分)