科目：**工程數學（電機電子組）**　　　　適用系所：**工業教育學系**

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. Solve the differential equation $x^2\dfrac{dy}{dx}=\dfrac{1}{2}x^2+y^2$. (10 分)

2. Solve the differential equation $x^2y''+xy'+4y=2\sin(2\ln(x))$. (10 分)

3. Find the general solution of the system. (15 分)

$$\frac{dx}{dt}=x-y+4z,\quad \frac{dy}{dt}=3x+2y-z,\quad \frac{dz}{dt}=2x+y-z$$

4. Suppose that $\mathbf{A}$ is a square matrix. Is it true that $\mathbf{AA}^T$ and $\mathbf{A}^T\mathbf{A}$ are orthogonally diagonalizable? Justify your answer. (10 分)

5. Find the rank and nullity of the matrix $\mathbf{A}$. (10 分)

$$\mathbf{A}=\begin{bmatrix} 2 & 9 & 6 & 5 & 4 \\ 6 & -1 & 4 & 1 & -2 \\ -2 & -1 & -2 & -1 & 0 \\ 4 & 8 & 7 & 5 & 3 \end{bmatrix}$$

6. Solve the initial value problem by using the Laplace transform. (15 分)

$$y''+4y'+4y=g(t);\ y(0)=1,\ y'(0)=2$$

$$g(t)=\begin{cases} 2 & \text{for } 0\le t<2 \\ 0 & \text{for } t\ge 2 \end{cases}$$

7. Evaluate $\oint_c 3e^z(z^2-4)^2(z+i)^{-2}dz$, where c: $|z-1+2i|=4$. (10 分)

8. Find the orthogonal projection of the vector $\mathbf{u} = (-1, 0, 1, 2)$ onto the subspace of

   R^4 spanned by the vectors $\mathbf{u}_1 = (2, 1, 2, -1)$, $\mathbf{u}_2 = (2, 2, 6, 0)$, $\mathbf{u}_3 = (3, 1, -1, -3)$. (10

   分)

9. Suppose that $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are vectors in R^n. Is it true that if $\mathbf{u}$ is orthogonal

   to $\mathbf{v}$ and $\mathbf{w}$, then $\mathbf{u}$ is orthogonal to $\mathbf{v} + \mathbf{w}$? Justify your answer. (10 分)