科目：**工程數學（精密機械組）**　　　　適用系所：**機電科技學系**

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

【試題 1】(15 分)

If $y = y(t)$, solve ODE: $y''' + 3y'' + 3y' + y = 0$,

(a) Find the general solution.

(b) Is $y(t)$ stable or unstable?

【試題 2】(15 分)

Please solve the following ODE equation using Laplace transform method (Shifted data problem).

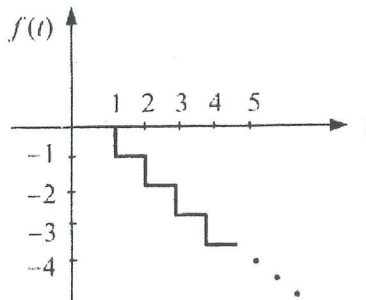$$y'' + 2y' - 3y = 0, \quad y(2) = -3, \quad y'(2) = -5$$

【試題 3】(15 分)

Find the Laplace transform of the following function.

(a)　　　　　　　　　　　　　　(b)

【試題 4】(20 分)

Given the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$
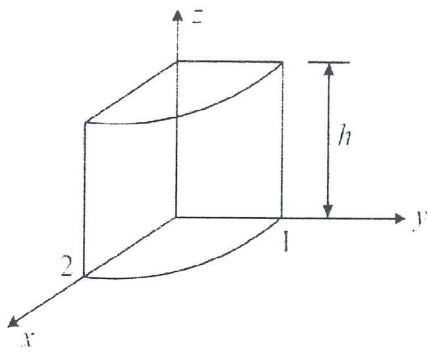
(a) Find a matrix S that diagonalizes the matrix A.

(b) Calculate e^{A}.

【試題 5】(20 分)

Compute the flux of water through the portion of the surface defined as
$S : x^2 + 4y^2 = 4, \ x \geq 0, \ y \geq 0, \ 0 \leq z \leq h$, if the velocity vector is
$\vec{v} = \vec{F}(x, y, z) = [y^3, \ x^3, \ z^3]$, speed being measured in m/s. (Generally,
$\vec{F} = \rho\vec{v}$, but water has the density $\rho = 1$)

(a) Use the method of parametric representation

(b) Use the method of y-z plane projection

【試題 6】(15 分)

(a) $f(x) = e^{-\alpha x}, \ \alpha > 0$, please find Fourier cosine integral representation;

(b) Using (a), find $\int_0^\infty \dfrac{\cos 3x}{x^2 + 9} dx$.