

國立臺灣師範大學 101 學年度碩士班招生考試試題

科目：機率

適用系所：應用電子科技學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. (10 分) In an experiment with equiprobable outcomes, the event space is $S = \{1, 2, 3, 4\}$. $P[s] = 1/4$ for all $s \in S$. Show that the three events $A_1 = \{1, 3, 4\}$, $A_2 = \{2, 3, 4\}$, and $A_3 = \emptyset$ are dependent.

2. (共 20 分) Let random variable X have the binomial probability mass function (PMF)

$$P_X(x) = \binom{4}{x} (1/2)^4.$$

(a) Find $E[X]$, the expected value of X (5 分).

(b) Find $\text{Var}[X]$, the variance of X (5 分).

(c) Given the condition $B = \{X \neq 0\}$, what is the conditional expected value $E[X|B]$ (5 分)? What is conditional variance $\text{Var}[X|B]$ (5 分)?

3. (共 25 分) The probability density function (PDF) of random variable X is

$$f_X(x) = \begin{cases} (1/2)e^{-x/2} & x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

(a) What is $P[X > 1]$ (5 分)?

(b) What is $E[X]$ (5 分)?

(c) What is $\text{Var}[X]$ (5 分)?

(d) Given $X > 1$, Find $E[X|X > 1]$ (5 分) and $\text{Var}[X|X > 1]$ (5 分).

4. (共 25 分) Random variables X and Y have the joint PMF

$$P_{X,Y}(x, y) = \begin{cases} cxy & x = 1, 2, 4; y = 1, 3, \\ 0 & \text{otherwise.} \end{cases}$$

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- (a) What is the value of the constant c (5 分)?
- (b) What is the marginal PMF $P_X(x)$ (5 分)?
- (c) What is the marginal PMF $P_Y(y)$ (5 分)?
- (d) Find $E[X]$ (5 分).
- (e) Find $E[Y]$ (5 分).

5. (共 20 分) Random variables X and Y have joint PDF

$$f_{X,Y}(x, y) = \begin{cases} (x + y)/3 & 0 \leq x \leq 1, 0 \leq y \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

Let $A = \{Y \leq 1\}$.

- (a) Find $P[A]$ (5 分).
- (b) Find $f_{X,Y|A}(x, y)$ (5 分).
- (c) Find $f_{X|A}(x)$ (5 分).
- (d) Find $f_{Y|A}(y)$ (5 分).