

# 國立臺灣師範大學 101 學年度碩士班招生考試試題

科目：工程數學

適用系所：應用電子科技學系

注意：1.本試題共1頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. (10 分) Find the general solution  $y'' - 2y' = e^x \sin x$ .
2. (12 分) Find the general solution  $(x+2)^2 y'' - (x+2)y' + y = 3x + 4$ .
3. (12 分) Find the general solution  $4xy + 6y^2 + (2x^2 + 6xy)y' = 0$ .

4. (16 分) Find the general solution 
$$\begin{cases} \dot{x} - x + \dot{y} + 2y = 1 + e^t \\ \dot{y} + 2y + \dot{z} + z = 2 + e^t \\ \dot{x} - x + \dot{z} + z = 3 + e^t \end{cases}$$

5. (15 分) A linear time-invariant system is

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t), \quad y(t) = [1 \quad 0] \mathbf{x}(t) + 2u(t)$$

where  $\mathbf{x}(t) = [x_1(t) \quad x_2(t)]^T$ . Solve  $y(t)$  if  $u(t)$  is a unit step function and

$$\mathbf{x}(0) = [2 \quad 0]^T.$$

6. (共 15 分) In  $\mathbf{C}([0,1])$ , let  $f(t) = t$  and  $g(t) = e^t$ . Compute the inner product

$$\langle f, g \rangle \equiv \int_0^1 f(t)g(t)dt \quad (5 \text{ 分}), \quad \|f\| \equiv \sqrt{\langle f, f \rangle} \quad (5 \text{ 分}), \quad \text{and} \quad \|f + g\| \quad (5 \text{ 分}).$$

7. (10 分) Find an orthonormal set from the vectors  $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ ,  $\mathbf{x}_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$  and  $\mathbf{x}_3 = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$ .

8. (共 10 分) Determine all eigenvalues (3 分), regular eigenvectors (3 分) and generalized eigenvectors (4 分) for the operator represented by the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ -3 & 3 & 5 \end{bmatrix}.$$