國立臺灣師範大學 101 學年度碩士班招生考試試題

科目：**應用數學**　　適用系所：**物理學系**

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. (a) What is the Stokes's theorem in vector analysis? (5 marks)

   (b) Verify the Stokes's theorem for $\vec{A} = -y\vec{\imath} + x\vec{\jmath} - xyz\vec{k}$ by considering the surface S consisting of the cone $z = \sqrt{x^2 + y^2}$ for $x^2 + y^2 \leq 9$. The normal vectors for the surface S are chosen to point inward (point "into" the cone). (15 marks)

   (c) Evaluate the following surface integral by using the Divergence theorem

   $$\iint_{S_1} \vec{B} \cdot d\vec{\sigma},$$

   where $\vec{B} = x^3\vec{\imath} + y^3\vec{\jmath} + z^3\vec{k}$, and S_1 is the surface of a sphere of radius R, with its center located at the origin . The normal vectors for the surface S_1 are chosen to point outward. (5 marks)

2. Given a matrix

   $$A = \begin{pmatrix} 0 & 1+i \\ 1-i & 0 \end{pmatrix},$$

   (a) Show that A is a Hermitian matrix. (5 marks)

   (b) Find the eigenvalues and the corresponding eigenvectors of the matrix A. Remember to normalize each eigenvector to a unit vector. (10 marks)

   (c) Show that the eigenvalues of a Hermitian matrix B are real. Further, show also that the eigenvectors of distinct eigenvalues of B, **considered as vectors in the complex space**, are orthogonal to each other. (10 marks)

(下一頁尚有題目，請翻頁後繼續作答)

3. Find the general solutions to the following ODEs.

(a) $y'(x) - 3y(x) = x^2$, (5 marks)

(b) $x^2y''(x) - xy'(x) + y(x) = x(ln(x))^2$, where ln is the notation for natural logarithm. (10 marks)

4. Consider the following second-order ordinary differential equation with the given initial conditions

$$y''(x) + xy'^{(x)} - y = 1 + x^2,\ y(0) = -1,\ y'(0) = 1.$$

(a) Let $y(x) = \sum_{n=0}^{\infty} a_n x^n$. Find the recurrence relation for $n \geq 3$? (10 marks)

(b) Using the method of power-series to write out the solution of this ODE explicitly up to the term of x^6. (10 marks)

5. Evaluate the following integral by using the residue theorem in complex analysis. (15 marks)

$$\int_0^{2\pi} \frac{1}{(5 - 3\cos\theta)^2} d\theta.$$