國立臺灣師範大學101學年度碩士班招生考試試題

科目：近代物理　　適用系所：物理學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. (A) Write down the four assumptions of the Bohr model of a hydrogen atom. Derive the energy of a hydrogen atom and the Bohr radius. [12 points]

(B) For the Balmer series of the hydrogen atom, find the longest wavelength of emitted photons in the series and determine their energy. [8 points]

2. For a particle moving in an infinite square well, $V(x) = 0$ for $|x|<L/2$ and $V(x) = \infty$ for $|x|\geq L/2$.

(A) Write down the time-dependent Schrödinger equation and time-independent Schrödinger equation (TISE). [4 points]

(B) For a particle in the ground state, the eigenfunction is $\sqrt{\frac{2}{L}}\cos(\frac{\pi}{L}x)$ obtained by solving the TISE. Derive the expectation values of x, x^2, p, and p^2. [12 points]

(C) Calculate the value of $\Delta x \Delta p$ from the result in 2(B). [4 points]

3. Electrons bound to metal are literally torn from the surface by the application of a strong electric field. This phenomenon is called field emission. By placing a positive charged plate near the source metal to form a parallel-plate capacitor, once beyond the surface ($x>0$), one electron is attracted by the electric force in the gap, $F=e\varepsilon$, represented by the potential energy $U(x)=-e\varepsilon x$.

(A) Starting from the equation for transmission $T(E)$ of electrons from high, wide barriers $T(E) \approx \exp(-\frac{2}{\hbar}\sqrt{2m}\int\sqrt{U(x)-E}dx)$, show that $T(E) \approx \exp(-\frac{\varepsilon_c}{\varepsilon})$ with $\varepsilon_c = \frac{4\sqrt{2m}\,|\phi|^{3/2}}{3e\hbar}$, where ϕ and ε_c denote the work function and the characteristic field strength, respectively. [10 points]

(B) With a typical value of ϕ=4.0 eV, the calculated value of ε_c is 5.5×10^{10} V/m. The number of electrons per second impinging on the plate surface from the bulk is the collision frequency f, about 10^{30} per second per square centimeter for most metals. Take the plate separation to be d=8.0 μm and plate area to be A=1.0 cm^2. Estimate the leakage current due to tunneling that passes across a parallel-plate capacitor charged to a potential difference of 10 kV. ($e^{-44}=7.78\times10^{-20}$) [10 points]

4. The electric field $\vec{E}_d$ due to an electric dipole is given by $\vec{E}_d = k[\frac{\vec{p}}{r^3} - \frac{3(\vec{p}\cdot\vec{r})}{r^5}\vec{r}]$, where $|\vec{p}|$=qa is the dipole moment. At a large separation, i.e., r >> a, assume that the magnitude of the electric force between two molecules is proportional to $\frac{1}{r^n}$.

(A) Determine the value n in the formula $\frac{1}{r^n}$ for a hydrogen bond. [10 points]

(B) Determine the value n in the formula $\frac{1}{r^n}$ for a van der Waals interaction. [10 points]

5. The figure show the total molecular energy for the bonding and antibonding orbitals of H_2. Based on the symmetry of the electron wave function for a H_2 system, explain the dependence of the molecular potential energy on the separation between the protons. [20 points]