國立臺灣師範大學101學年度碩士班招生考試試題

科目：**基礎數學**　　適用系所：**科學教育研究所**

注意：1.本試題共1頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

Fundamental Caculus

1. (10 points) Evaluate the integral

$$\int_0^1 \tan^{-1} x dx.$$

2. (10 points) Evaluate the double integral

$$\int_0^1 \int_y^1 e^{-x^2} dx dy.$$

3. (10 points) Determine whether the following series converges:

$$\sum_1^\infty \frac{\tan^{-1} k}{k^2+1}.$$

4. (10 points) Show that $|\cos x - \cos y| \leq |x - y|$ for all real numbers x and y.

5. (10 points) The radius of a sphere is measured with a percentage error within $\pm 0.04\%$. Estimate the percentage error in calculated volume of the sphere. The volume V of a sphere is $V = \frac{4}{3}\pi r^3$.

Linear Algebra

1. (15 points) Diagonize the matrix

$$\begin{bmatrix} 3 & 1 \\ -3 & 7 \end{bmatrix}.$$

2. (15 points) Let matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 & 4 \\ 1 & 2 & 1 & 1 & 6 \\ 0 & 1 & 1 & 1 & 3 \\ 2 & 2 & 0 & 1 & 7 \end{bmatrix}.$$

Determine the nullspace $N(A)$.

3. Suppose B is an $n \times n$ real matrix with the property that $B^2 = B$.
   (a) (10 points) Show that if λ is an eigenvalue of B, then $\lambda = 1$ or $\lambda = 0$.
   (b) (10 points) Determine whether B is diagonalizable.