科目：**工程數學**   適用系所：**光電科技研究所**

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

第 1 題到第 4 題為選擇題，不倒扣，不須寫出求解過程，作答時請標明題號。

第 5 題到第 10 題，須寫出求解過程及答案，作答時請標明題號。

1. Solve the given equation $\int_0^t f(\tau)f(t-\tau)d\tau = 6t^3$.   **(5 point)**

   (A) $(t) = \pm\sqrt{6}t$,   (B) $f(t) = \pm\sqrt{18}t$,   (C) $f(t) = \pm18t$,

   (D) $f(t) = \pm36t$,   (E) $f(t) = \pm6t$

2. Use Laplace transform to solve $xy'' + (1-x)y' - ky = 0$.   **(5 point)**

   (A) $y = \frac{e^t}{t!}\frac{d^k}{dt^k}[t^k e^{-t}]$,   (B) $y = \frac{e^t}{k!}\frac{d^k}{dt^k}[t^k e^t]$,   (C) $y = \frac{e^t}{k!}\frac{d^k}{dt^k}[t^k e^{-t}]$,

   (D) $y = \frac{e^t}{k}\frac{d^k}{dt^k}[t^k e^{-t}]$,   (E) $y = \frac{e^t}{k}\frac{d^k}{dt^k}[t^k e^t]$

3. Fourier transform $F\left\{exp\left(-\frac{x^2}{4p^2}\right)\right\} = ?$ **(5 point)**

   (A) $2\sqrt{\pi}\, p\, exp(-4p^2\alpha^2)$,   (B) $2\sqrt{\pi}\, p\, exp(-p^2\alpha^2)$,   (C) $2\sqrt{\pi}\, p\, exp\left(\frac{-\alpha^2}{4p^2}\right)$,

   (D) $2\sqrt{\pi}\, p\, exp\left(\frac{-p^2\alpha^2}{4}\right)$,   (E) $2\sqrt{\pi}\, p\, exp\left(\frac{\alpha^2}{4p^2}\right)$

4. Find the inverse Laplace Transform $\frac{s-6}{(s-1)^2+4} = ?$ **(5 point)**

   (A) $e^t\left(cos t + \frac{5}{2}sin2t\right)$,   (B) $e^t\left(cos t - \frac{5}{2}sin2t\right)$,   (C) $e^{-t}\left(cos t + \frac{5}{2}sin2t\right)$,

   (D) $e^t\left(cos2t + \frac{5}{2}sin2t\right)$,   (E) $e^t\left(cos2t - \frac{5}{2}sin2t\right)$

5. Solve the initial value problem for the following Ordinary differential equation.

   $y'' + y = 5x + 8sinx,\ y(\pi) = 0,\ y'(\pi) = 2$ **(10 point)**

6. Use the Laplace transform to solve differential equation:

    $ty'' + (4t - 2)y' - 4y = 0; \ y(0) = 1$ **(10 Point)**

7. Evaluate the following integrals by complex function theory $\int_0^\infty \dfrac{1}{\sqrt{x}(x+4)(x-5)} dx$.

    (10 Point)

8. Consider a matrix equation $A=BC$, find matrix C, if $A = \begin{bmatrix} 1 & 2 & 7 & 4 \\ 5 & 6 & 15 & 8 \\ 9 & 10 & 23 & 12 \end{bmatrix}$;

    $B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$ **(10 point)**

9. When $A = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}$, calculate $\ln(I+A)$. Note: ln is defined by the Taylor's expansion of the logarithm. **(20 point)**

10. For the quadric from $Q = 5x_1^2 - 8x_1 x_2 + 5x_2^2 = 36$, determine what type of conic section is represented and transform it to principle axes. Express $X^T = [x_1, x_2]$ in term of the new coordinate vector. $Y^T = [y_1, y_2]$. **(20 point)**