1. [15 points] Compute the determinant for $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 319 & 193 & 0 \\ 1 & 19 & 931 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$.

2. [20 points] Let $A = \begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}$. (a) Find the eigenvalues and the corresponding eigenvectors. (b) Find A^n.

3. [15 points] Determine the value of t for which the following system has a unique solution and show the solution.

    $3tx - 2y = 4$
    $-6x + ty = 1$

4. [20 points] A battery of 9 volts is applied to the terminals of a resistor, which was selected from a bunch of resistors with tolerance of 10 ± 1 ohms. Let X be the value of resistor with probability density function $f(x) = c$. (a) Find the value of c. (b) What will be the expected value of the current in the resistor?

5. [15 points] If X and Y are independent, normal random variables with $E(X) = 0$, $Var(X) = 4$, $E(Y) = 10$, and $Var(Y) = 9$. Find the following values: (a) $E(2X+3Y)$ (b) $Var(2X+3Y)$.

6. [15 points] Customers arrive in a certain shop according to a Poisson process at a mean rate of 20 per hour. What is the probability that the waiting time until the first arrival is more than 6 minutes?

1. [15 points] Compute the determinant for $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 319 & 193 & 0 \\ 1 & 19 & 931 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$.

2. [20 points] Let $A = \begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}$. (a) Find the eigenvalues and the corresponding eigenvectors. (b) Find A^n.

3. [20 points] Determine the value of t for which the following system has a unique solution and show the solution.

$$3tx - 2y = 4$$
$$-6x + ty = 1$$

4. [25 points] Solve the initial value problem by convolution

$$y'' + 5y' + 6y = r(t), \quad r(t) = \begin{cases} 0, & for\ t < 2 \\ 1, & for\ 2 < t < 4, \quad y(0) = 0,\ y'(0) = 0. \\ 0, & for\ t > 4 \end{cases}$$

5. [20 points] Solve the initial value problem.

$$y_1' = 3y_1 - 4y_2 + 20\cos t$$
$$y_2' = y_1 - 2y_2$$
$$y_1(0) = 0,\ y_2(0) = 8$$