大同大學 101 學年度研究所碩士班入學考試試題

考試科目:工程數學　　所別:光電工程研究所　　

註:本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算

1. Find the general solution of the following differential equations

(a) $x^2y'' - 2xy' + 2y = x\ln x$

(b) $(x - \frac{y}{y'})(y - xy') = 2$

(20 %)

2. Given the governing equation

$$D\frac{\partial^2 C}{\partial x^2} = \frac{\partial C}{\partial z}$$

with the following boundary conditions (BC):

BC_1: $C = C_0$ at $z = 0$, for all x;

BC_2: $C = C_1$ at $x = 0$, for $z > 0$;

BC_3: $C = C_0$ at $x = \infty$, for all z;

(a) Solve by the combination of variables technique.

(b) It is interesting to note that if z is replaced by t, an unsteady-state partial differential equation results. Sovle by Laplace transform technique. (20 %)

3. Find the general solution of each of the following equation:

(a) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = -\exp(2x)/(\exp(x) + 1)$

(b) $x^4\frac{d^4y}{dx^4} + 6x^3\frac{d^3y}{dx^3} + 9x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = (1 + \ell nx)^2$

(20 %)

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4.

Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$,

(a) Find all the eigenvalues of A

(b) Find all the eigenvectors of A associated with each eigenvalue of A

(c) Find an invertible matrix P so that $P^{-1}AP$ is diagonal

(d) Compute A^{100} (20 %)

5. Assume A and B are two $n \times n$ square matrices, please show that

$$\det\left(\begin{bmatrix} A & B \\ B & A \end{bmatrix}\right) = \det(A+B)\det(A-B)$$

(20 %)