| 科 目 | 微積分 | 適用系別 | 財務工程與精算學士學位學程 二年級 | 時間 | 80分鐘 |
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1. (a) Evaluate $\lim\limits_{x \to 1} f(x)$ if $f(x) = \begin{cases} 3-2x & , x < 1 \\ 1 & , x = 1 \\ 2+\sqrt{x-1} & , x > 1 \end{cases}$.　　(16%)

(b) Is $f$ continuous at $x = 1$? (Explain why)

(c) Is $f$ differentiable at $x = 1$ ? (Explain why)

(d) Sketch the graph of $f$.

2. (a) Find $y'$, if $x^3 + y^3 = 6xy$　　　　　　　　(12%)

(b) Find the equation of the tangent line to the curve $x^3 + y^3 = 6xy$ at the point **(3,3)**

3. Find the derivative for the following functions:　　　　(12%)

(a) $G(x) = \int_{e^x}^{x^3} \sqrt{1+t^2}\, dt$　　(b) $f(x) = \left(e^{\cos x^2}\right)\left(\log_{10}\left(x^2 - 4\right)\right)$

4. Evaluate (a) $\int_0^1 \ln x\, dx$　　(b) $\int x\sqrt{x+1}\ dx$　　　　　　(12%)

5. Evaluate $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{2+x^3}\ dxdy$ by reversing the order of integration　　(12%)

6. Evaluate the following limits: (12%)

(a)　$\lim\limits_{n \to \infty} \sum\limits_{k=1}^{n} \dfrac{1}{k(k+1)}$　　　　　　(b)　$\lim\limits_{x \to 0} \dfrac{e^x - 1}{\sin x}$

7. Find the area bounded by the two functions, $f(x) = 2 - x^2$ and $g(x) = -x$. (12%)

8. A function $f : R \to R$ is defined as $f(x) = x^3 + ax^2 + bx + c$. If $f(x)$ has local maximum at $x = 1$ and $f(x)$ has local minimum at $x = 3$, find a and b. (12%)