

逢甲大學103學年度轉學生招生考試試題

編號：轉031-3

科目	微積分	適用 系別	二年級(一)組群、(四)組群、 (五)組群、(六)組群、應用數學 系	時間	80分鐘
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※ 請務必在答案卷作答區內作答 ※ 共二頁第一頁

一、是非與填充題 (60%) [共12格，每格5分，不用列出計算過程]

(A) 1~5題為是非題，請依續作答於答案卷上。若認為敘述正確者，請填圈 \bigcirc ；若認為敘述錯誤者，請填叉 \times 。

1. () If $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$ diverges (發散), then $\sum_{n=1}^{\infty} a_n^2 = a_1^2 + a_2^2 + a_3^2 + \dots$ also diverges.

2. () If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges (收斂).

3. () The value of the definite integral $\int_{-1}^1 \left(x^2 + \frac{x \cos x}{1+x^2+x^4} \right) dx$ is $\frac{2}{3}$.

4. () If $\lim_{x \rightarrow a} [f(x) - g(x)]$ exists, then $\lim_{x \rightarrow a} f(x)$ exists and $\lim_{x \rightarrow a} g(x)$ exists.

5. () The value of the limit $\lim_{x \rightarrow 2} \frac{x^{10} - 2^{10}}{x^5 - 2^5}$ is 64. (Hint: Use the derivative)

(B) 6~12題為填充題，請註明題號依續作答於答案卷上。

6. Let $f(x) = \begin{cases} \frac{\sin 2x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$. If $k = \underline{\hspace{2cm}}$, then f is continuous everywhere.

7. $\frac{d}{dx} \left(\frac{x^3 - 4x^2 + 3}{\sqrt{x}} \right) = \underline{\hspace{2cm}}$.

8. Let $f(x) = \begin{cases} -x+1, & x \leq 0 \\ 2x^2+1, & x > 0 \end{cases}$. Then $\int_{-1}^1 f(x) dx = \underline{\hspace{2cm}}$.

$$9. \int \frac{\sin(2 + \frac{1}{x})}{x^2} dx = \underline{\hspace{2cm}}.$$

$$10. \frac{d}{dx} (x + \sqrt{x})^{\frac{1}{x}} = \underline{\hspace{2cm}}.$$

$$11. \frac{\partial}{\partial x} \int_{x^2}^{\ln y} te^{-t} dt = \underline{\hspace{2cm}}.$$

$$12. \text{The sum of the series } \sum_{n=1}^{\infty} \frac{2}{n(n+1)(n+2)} = \frac{2}{1 \cdot 2 \cdot 3} + \frac{2}{2 \cdot 3 \cdot 4} + \frac{2}{3 \cdot 4 \cdot 5} + \dots \text{ is}$$

$$S = \underline{\hspace{2cm}}. \quad (\text{Hint: } \frac{2}{k(k+1)(k+2)} = \frac{1}{k(k+1)} - \frac{1}{(k+1)(k+2)})$$

二、計算題 (40%) [每一大題 10 分；請寫下詳細推導、計算過程，否則不予計分]

1. Find $f(x)$, if $f''(x) = 20x^3 - 18x$, and $f'(1) = f(0) = -3$.

2. Find y' by logarithmic differentiation, if $y = \frac{(x^3 - 1)^4 \sqrt{3x - 1}}{x^2 + 4}$, $x > 1$.

3. (1) Sketch the region bounded by the graphs of $y = e^x$ and $y = x$ and the lines $x = 0$ and $x = 1$.

(把這些圖形畫出來，並標示它們所圍住的區域)

(2) Find the area of the region in (1).

4. Find the volume of the solid lying under the surface $z = x^3 + 4y$ and above the region R in the xy -plane bounded by the line $y = 2x$ and the parabola $y = x^2$.