MASTER PROGRAM ENTRANCE EXAMANATION–GENERAL PHYSICS

(1)

(5%) A man drops a baseball from the edge of a roof of a building. At exactly the same time another man shoots a baseball vertically up towards the man on the roof in such a way that the ball just barely reaches the roof. Does the ball from the roof reach the ground before the ball from the ground reaches the roof, or is it the other way around?

(2)

(10%) Hold a basketball in one hand, at a height of 1 m (see Fig. 1). Hold a baseball in the other hand about two inches above the basketball. Drop them simultaneously onto a hard floor. The basketball will rebound and collide with the baseball above it. Estimate the rebound speed of the baseball? Assume that the basketball is three to four times heavier than the baseball.

(3)

(15%) In Fig. 2, two objects of mass m_1 and m_2 are connected by a single spring whose spring constant is k and whose un-stretched equilibrium length is L. We assume the objects move horizontally with no friction or damping due to the environment. Applying Newton's Second Law of Motion to each object, show that

$$m_1 \frac{d^2 x_1}{dt^2} = k\left(x_2 - x_1 - L\right) \text{ and } m_2 \frac{d^2 x_2}{dt^2} = -k\left(x_2 - x_1 - L\right). \text{ (10\%)}$$

Let $z = x_2 - x_1 - L$, then show that

$$\frac{d^2 z}{dt^2} = -\frac{k}{m} z$$

where the *reduced mass m* is defined by

$$m = \frac{m_1 m_2}{m_1 + m_2}. \text{ (5\%)}$$

(4)

(5%) When an object is heated and expands, what happens to any holes in the object? Do they get larger or smaller?

(5)

(10%) Newton's Law of Cooling states that the rate dT/dt at which the temperature $T=T(t)$ of an object changes with respect to time t, is proportional to the difference $T_A - T$ between the ambient temperature T_A of the environment, and the temperature T of the object; that is

$$\frac{dT}{dt} = k\left(T_A - T\right) \qquad (1)$$

where $k > 0$ is a positive real constant.

1. Find the solutions to this equation that correspond to $T(0)=10^\circ$, $T(0)=60^\circ$. (4%)
2. If $T(0) < T_A$:
   a. what does Eq. (1) imply about the sign of dT/dt? (1%)
   b. does this mean T is increasing or decreasing? (1%)
   c. explain (on physical grounds) why T should approach A as t gets large. (1%)
3. If $T(0) > T_A$:
   a. what does Eq. (1) imply about the sign of dT/dt? (1%)

b. does this mean T is increasing or decreasing? (1%)

c. explain (on physical grounds) why T should approach A as t gets large. (1%)

(6)

(15%) Fig. 3 represents a model for a thermodynamic cycle of a *Stirling engine*. Fuel is burned externally to warm one of the engine's two cylinders. A fixed quantity of inert gas moves cyclically between the cylinders, expanding in the hot one and contracting in the cold one. Consider n mol of an ideal monatomic gas being taken once through the cycle, consisting of two isothermal processes at temperatures $3T_i$ and T_i and two constant-volume processes.

1. Determine, in terms of n, R, and T_i, the net energy transferred by heat to the gas.
2. Determine, in terms of n, R, and T_i, the efficiency of the engine.

Where R= 8.314 J/mol·K is the *universal gas constant*.

(7)

(15%) Fig. 4 shows a disk of radius R with a uniform surface charge density σ.

1. Calculate the electric field **E**, in terms of R, σ, and ε_0 (permittivity of free space), at a point P that lies along the central perpendicular axis of the disk and a distance z from the center of the disk. (5%)
2. Show that the $R\to\infty$ limit yields $\sigma/(2\varepsilon_0)$, the value of the electric field due to an infinite plane of charge. (2%)
3. Show that the lowest order term in a small z expansion is the same as the previous result, indicating that very near the disk the field is approximately that of an infinite plane. (3%)
4. Expand the result for the magnitude of **E** as a power series in R and show that the lowest-order term is

$$E = \frac{Q}{4\pi\varepsilon_0 z^2},$$

what you would expect for a point charge. Where $Q=\sigma\pi R^2$ is the total charge of the disk. (5%)

(8)

(10%) Suppose a conductor of arbitrary shape contains a cavity as shown in Figure 5. Let us assume that no charges are inside the cavity. Prove that the electric field inside the cavity must be zero regardless of the charge distribution on the outside surface of the conductor.

(9)

(5%) Fig. 6 shows a square loop (length=width=2m) with resistance 1Ω (ohm=kg·m^2/A^2·s^3) moves at a constant velocity of 1 m/s to the right across two regions of uniform magnetic field B=1T (tesla=kg/A·s^2). One region extends from x =–4m to x=0m, and has a uniform field directed into the page. The second region extends from x=0m to x=4m, with a uniform magnetic field directed out of the page. The magnitudes of the fields are equal. Plot the magnetic flux through the loop, and the induced current, as a function of time. Take the flux to be positive if the field is into the page, and take clockwise to be positive for induced current.

(10)

(10%) The current in the solenoid shown in Fig. 7 is decreasing. A conducting loop is placed around the solenoid as shown. Explain that the induced current will flow in the loop in *clockwise* (CW) direction (viewed from left), as long as I_{sol} is changing as indicated in the problem.

Figures:

Fig. 1

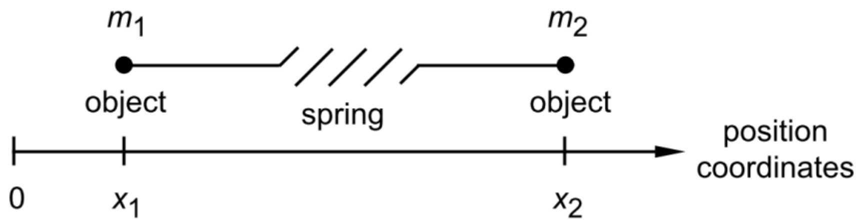

Fig. 2

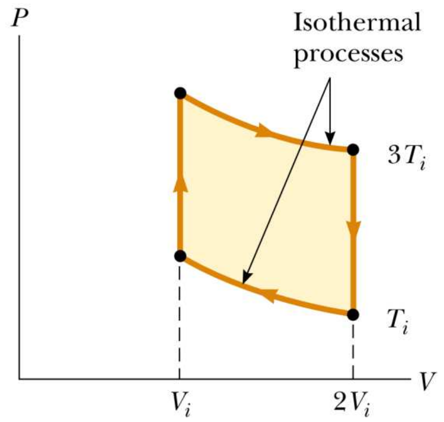

Fig. 3

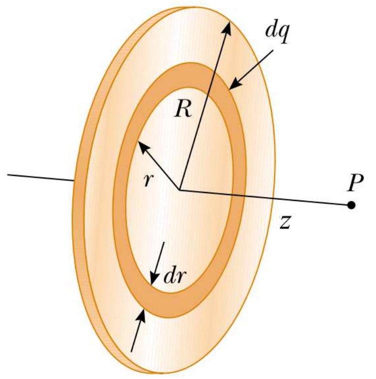

Fig. 4

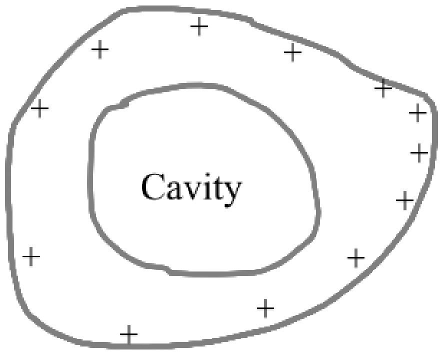

Fig. 5

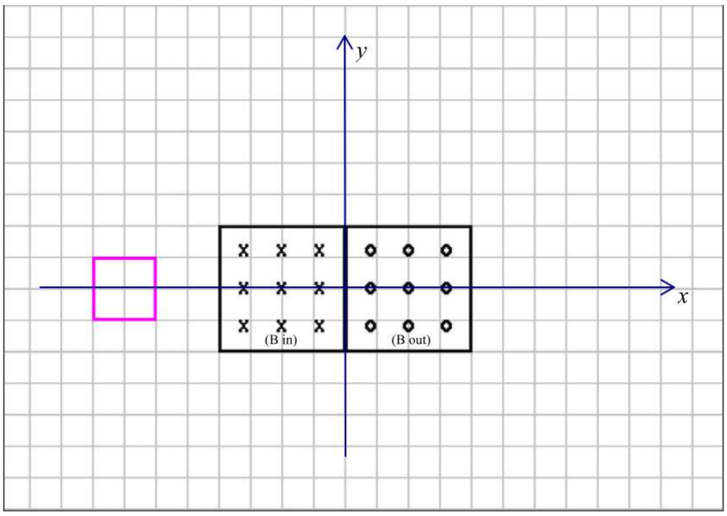

Fig. 6

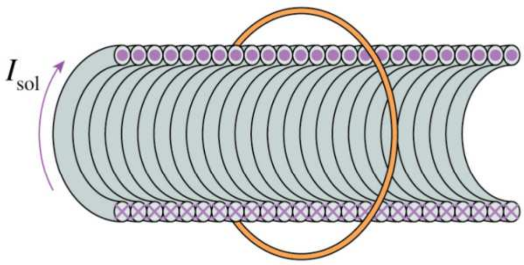

Fig. 7