科目	線性代數與機率	適用系所	通訊工程學系	時間	100 分鐘

※請務必在答案卷作答區內作答。

1. (10%) Two dices are rolled until at least one die lands on 1 or sum up 7, then we stop the rolling. Let random variable X be the number of times that we rolled. What is the expected value of X.
2. (15%) The number of times, denoted by random variable X, that a person contacts a cold in a given year is a Poisson random variable with parameter $\lambda = 6$.
   a. Find that the expected value and variance of the random variable of X.
   b. Suppose that a new wonder drug has just been marketed that reduces the Poisson parameter to $\lambda = 2$ for 75 percent of the population. For the other percent 30 percent of the population the drug has no effect on colds. If a person tries the drug for a year and has 2 colds in that time, what is the probability that the drug is beneficial for him or her?

   Hint: The probability mass function of the Poisson random variable with parameter λ is given by $P\{X = i\} = e^{-\lambda}\dfrac{\lambda^i}{i!}$.
3. (15%) The jointly probability density of X and Y is

$$f(x,y) = \begin{cases} k(x+y) & 0 < x < 1, \quad 0 < y < 1 \\ 0 & otherwise \end{cases}$$

   Find the value of k.
   a. Are X and Y are independent.
   b. Find the Marginal density function, $f_X(x)$ and $f_Y(y)$.
   c. Find $P[X + Y < 1]$.
4. (10%) If X uniformly distributed over (-2,2), find
   a. The probability. $P[|X| > 1]$.
   b. The probability density function of $Y = |X|$.

5. (6%) Please find the inverse of $\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$.

6. (8%) For a matrix $\mathbf{A} = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & -1 & -3 & 1 & 3 \\ -2 & -1 & 1 & -1 & 3 \\ 0 & 3 & 9 & 0 & -12 \end{bmatrix}$. Please find the rank and nullity of $\mathbf{A}$.

7. Let $B = \left\{ \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix} \right\}$ be a basis for an inner product space S.

   (a) (10%) Please find an orthonormal basis for S using Gram-Schmidt orthonormalization process.

   (b) (5%) Please find the orthogonal complement of S.

8. For a matrix $\mathbf{A} = \begin{bmatrix} 2 & 2 & -2 \\ 2 & -1 & 4 \\ -2 & 4 & -1 \end{bmatrix}$.

   (a) (6%) Please find the eigenvalues of the matrix $\mathbf{A}$.

   (b) (15%) Please find the eigenvalues and eigenvectors of the matrix $\mathbf{A}^3$.