逢甲大學101學年度碩士班招生考試試題 編號：061 科目代碼：

科目	離散數學	適用系所	資訊工程學系	時間	100 分鐘

※請務必在答案卷作答區內作答。

1. (10 %) Please determine the truth value of each of these statements if the domain for all variables consists of all integers.
   (a) $\forall n \exists m (n^2 < m)$
   (b) $\exists n \exists m (n^2 + m^2 = 5)$.
   (c) $\exists n \exists m (n + m = 4 \wedge n - m = 1)$
   (d) $\forall n \forall m \exists p (p = (m+n)/2)$
2. (10%) Find these values.
   (a) Let $f(x) = \lfloor x^2/3 \rfloor$. Find $f(S)$ if $S = \{0,1,2,3,4,5\}$.
   (b) Let $S = \{-1,0,2,4,7\}$. Find $f(S)$ if $f(x) = \lceil x/5 \rceil$.
   (c) Let $f(x) = ax + b$ and $g(x) = cx + d$, where a,b,c,d are constants. Determine for which constants a,b,c,d it is true that $f \circ g = g \circ f$.
3. (10%) How many solutions are there to the equation
   $$x_1 + x_2 + x_3 + x_4 = 17$$
   where x_1, x_2, x_3, x_4 are nonnegative integers ?
4. (10%) (a) What is the coefficient of $x^{12}y^{13}$ in the expansion of $(x+y)^{25}$?
   (b) Show that if n is an nonnegative integer then $\sum_{k=0}^{n} 3^k \binom{n}{k} = 4^n$.
5. (10%) Find the solution to $a_n = 5a_{n-2} - 4a_{n-4}$ with $a_0 = 3$, $a_1 = 2$, $a_2 = 6$, and $a_3 = 8$.
6. (10%) Prove or disprove that there are three consecutive odd positive integers that are primes, that is odd primes of the form p, p+2, and p+4.
7. (10%) Calculate the maximum number of regions divided by n lines in the plane.
8. (10%) A bus driver pays all tolls, using only nickels (5cents) and dimes(10cents), by throwing one coin at a time into the mechanical toll collector.
   (a) Find a recurrence relation for the number of different ways the bus driver can pay a toll of n cents (where the order in which the coins are used matter).
   (b) In how many different ways can the driver pay a toll of 45 cents?
9. (10%) Show that a simple graph G with n vertices is connected if it has more than $(n-1)(n-2)/2$ edges.
10. (10%) Suppose that e is an edge in a weighted graph that is incident to a vertex v such that the weight of e does not exceed the weight of any other edge incident to v. Show that there exists a minimum spanning tree containing this edge.