逢甲大學101學年度碩士班招生考試試題 編號：044 科目代碼：

科目	微積分	適用系所	應用數學系A組、B組	時間	100 分鐘

※請務必在答案卷作答區內作答。

一、選擇題(40 %)：

1.Which of the following equality is false?

(A) $\lim_{x\to 1}\frac{x^{100}-1}{x-1}=100$ (B) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\sin(\frac{i}{n}\pi)=\frac{2}{\pi}$ (C) $\lim_{k\to\infty}\frac{k^3}{(\ln k)^2}=\infty$ (D) $\lim_{t\to 0}(1+3t)^{3/t}=e$.

2.The tangent line to the curve $x^y+y^x=3$ at the point $(2,1)$ is

(A) $(x-2)-2(1+\ln 2)(y-1)=0$ (B) $(x-2)+2(1+\ln 2)(y-1)=0$

(C) $2(1+\ln 2)(x-2)-(y-1)=0$ (D) $2(1+\ln 2)(x-2)+(y-1)=0$

3.Suppose that $f(x)=(x^2-1)^{1/3}/x$. Then $\{x \mid x \text{ is a critical value of } f\}=$

(A) $\{-1,1,0\}$ (B) $\{-\sqrt{3},\sqrt{3}\}$ (C) $\{-\sqrt{3},\sqrt{3},-1,1\}$ (D) $\{-\sqrt{3},\sqrt{3},-1,1,0\}$.

4.Suppose that g is the inverse function of a differentiable function f and $G=\frac{1}{g}$.

If $f(3)=0$ and $f'(3)=2$ then $G'(0)=$ (A) $-\frac{1}{18}$ (B) $\frac{1}{18}$ (C) $-\frac{2}{9}$ (D) $\frac{2}{9}$.

5.Which of the following equality is false?

(A) $\int_{-\pi/4}^{\pi/4}\csc x dx=0$ (B) $\int_0^1 3^{-x}dx=\frac{2}{3\ln 3}$ (C) $\int_{-1}^{1}\frac{1}{1+x^2}dx=\frac{\pi}{2}$ (D) $\int_1^{10}\log_{10}x dx=10-\frac{9}{\ln 10}$.

6.Which of the following series is divergent?

(A) $\sum_{n=1}^{\infty}\frac{e^n}{n!}$ (B) $\sum_{n=1}^{\infty}n\tan(\frac{1}{n^2})$ (C) $\sum_{n=1}^{\infty}\frac{3^n}{6^n+2^n}$ (D) $\sum_{n=1}^{\infty}\frac{\sqrt{2n+3}}{n^2-2n+4}$

7.Suppose that $\sum_{n=1}^{\infty}a_n$ and $\sum_{n=1}^{\infty}b_n$ are divergent series. Which of the following statement is true?

(A) $\lim_{n\to\infty}|a_n|\neq 0$. (B) $\sum_{n=1}^{\infty}|a_n|^2$ diverges. (C) $\sum_{n=1}^{\infty}\sqrt{|a_n|}$ diverges. (D) $\sum_{n=1}^{\infty}|a_n+b_n|$ diverges.

8.The radius of convergence of $\sum_{n=0}^{\infty}\frac{4^n}{n+1}(x-3)^{2n}=$ (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 2 (D) 4.

9. $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(x-\frac{\pi}{2})^{2n}=$ (A) e^x (B) $\tan^{-1}x$ (C) $\cos x$ (D) $\sin x$.

10.If $D=\{(x,y) \mid x^2+y^2\le 4, y\ge 0\}$ then $\iint_D\frac{x^2y}{\sqrt{x^2+y^2}}dA=$ (A) $\frac{8}{9}$ (B) $\frac{4}{3}$ (C) $\frac{16}{9}$ (D) $\frac{8}{3}$.

二、計算證明題 (60 %)：

1. **(12 %)** Evaluate a) $\frac{d^2}{dx^2}(\int_{x^3}^{1}\sqrt{t}\tan t dt)$ b) $\int_0^{\infty} e^{-x}\cos x\,dx$

2. **(12 %)** Let $f(x,y)=\begin{cases}\frac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$.

   a)Prove that $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

   b)Find $f_x(0,0)$ and $f_y(0,0)$.

3. **(12 %)** Suppose that f satisfies the inequalities $e^x \le f(x) \le e^x + x^2$, $|x|<1$.
   Find $f(0)$ and $f'(0)$.

4. **(12 %)** Suppose that $g(u,v)=u\ln(u+2v)$ and $f(x,y)=g(e^x+\sin\frac{y}{2}, e^x+\cos\frac{y}{2})$.

   Find a) $\nabla f(0,0)$.

   b)The tangent plane of $z=f(x,y)$ at the point $(0,0,\ln 5)$.

   c)The directional derivative of $f(x,y)$ at the point $(0,0)$ in the direction $<2,-1>$.

5. **(12 %)** Evaluate a) $\int_0^1\int_{x^2}^1 \cos(\sqrt{y})dydx$

   b) $\iint_D \frac{\ln(x^2+y^2)}{\sqrt{x^2+y^2}}dA$ where $D=\{(x,y):1\le x^2+y^2\le e\}$.