

科目	線性代數	適用系所	應用數學系A組	時間	100 分鐘
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※請務必在答案卷作答區內作答。

1. (20%) Consider $T: R^4 \rightarrow R^5$ is linear transformation defined by
 $T(a, b, c, d) = (a + 2b + 2d, 3a + 6b + 6d, -2a - 5b + 5c,$
 $-2b + 10c + 8d, -3b + 15c + 18d)$
 Find the rank and nullity of T .

2. (20%) Evaluate the determinant of matrix $A = \begin{bmatrix} 0 & 3 & 3 & 3 \\ 3 & 0 & 3 & 3 \\ 3 & 3 & 0 & 3 \\ 3 & 3 & 3 & 0 \end{bmatrix}$.

3. (20%) Consider $T: P_1(R) \rightarrow R^3$ is linear transformation, and
 $T(1+2x) = (7, 2, 7), T(-1+3x) = (8, -2, 13)$
 Find $T(a+bx)$.

4. (20%) Consider $A = \begin{bmatrix} 0 & -2 & -3 \\ -1 & 1 & -1 \\ 2 & 2 & 5 \end{bmatrix}$,

- (1) Determine all the eigenvalues of A .
 (2) For each eigenvalue λ of A , find the set of eigenvectors corresponding to λ .

5. (20%) Let $u_1 = (1, 1, 1), u_2 = (-1, 0, -1),$ and $u_3 = (-1, 2, 3),$ then $\{u_1, u_2, u_3\}$ is linear independent,

- (1) Apply the Gram-Schmidt process to $\{u_1, u_2, u_3\}$ to obtain the orthogonal vectors $\{v_1, v_2, v_3\}$.
 (2) Find the QR -factorization of matrix $A = [u_1 | u_2 | u_3]$.