逢甲大學101學年度碩士班招生考試試題 編號：019 科目代碼：

科目	統計學	適用系所	經濟學系、國際貿易學系、財稅學系丙組、科技管理研究所、合作經濟學系	時間	100 分鐘

※請務必在答案卷作答區內作答。

1. $X_1, X_2, \ldots, X_n$ are sampling from $N(\mu, \sigma^2)$. Considering the following point estimation of μ:

$$\hat{\mu}_1 = \frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X}$$

$$\hat{\mu}_2 = \frac{X_1}{2} + \frac{1}{2(n-1)}(X_2 + X_3 + \ldots + X_n)$$

(1) Which one is unbiased? Why? (5%)
(2) Which one is consistent? Why? (5%)
(3) Someone say that "*Unbiasedness is necessarily consistency, but consistency is not unnecessarily unbiasedness.*" Do you agree with this point? Why? (5%)

2. If $X_1, X_2, \ldots, X_n$ are sampling from $N(\mu, \sigma^2)$. To test H_0: $\mu \leq 30$ against H_1: $\mu > 30$, we reject H_0 and reject H_1 if and only if the sample mean $\bar{X} \geq 32$. If $n = 81$, $\sigma = 18$, then
(1) Find the probability of type 1 error if $\mu = 29$, and explain it. (7%)
(2) Find the probability of type 2 error if $\mu = 34$, and explain it. (7%)

3. Let X and Y have a bivariate normal distribution with $\mu_X = 80$, $\sigma_X^2 = 144$, $\mu_Y = 90$, $\sigma_Y^2 = 225$, and $\rho = 0.3$
Referring to the normal density table in the appendix, find the followings:
(1) E(X|Y = 72) (7%)
(2) Var(X|Y = 72) (7%)
(3) P(X ≤ 81|Y = 72) (7%)

4. (20%) One hundred students in the same class took the final exam and their teacher wants to evaluate whether the passing rates depend on the gender. The outcome that the students score a passing grade is listed in the table below.

	Boy	Girl	Total
Pass	36	24	60
Fail	24	16	40
Total	60	40	100

a. Find the probability that a student would pass the exam given he is a boy.

b. Let A be the event that the students fail the exam and let the B be the event that a female student is selected. Are the event A and event B independent? Why?

5. (15%) Let Y_1, Y_2, and Y_3 be independent, identically distributed random variables from a population with mean μ and variance σ^2. Denote $\bar{Y} = \frac{1}{3}\sum_{i=1}^{3} Y_i$ and

$$Z = \frac{2}{3}Y_1 + \frac{1}{6}Y_2 + \frac{1}{6}Y_3.$$

a. Find the expected value and variance of $\bar{Y}$

b. Is Z an unbiased estimator of μ? Which estimator ($\bar{Y}$ or Z) is a better estimator of μ? Why?

6. (15%) Suppose we use a simple linear regression model to study the relationship between the crime rate (%) and average alcoholic beverage consumption (*alcohol*, in kilograms) across the regions in a given year. Using observations from 50 areas, we obtains the following equation:

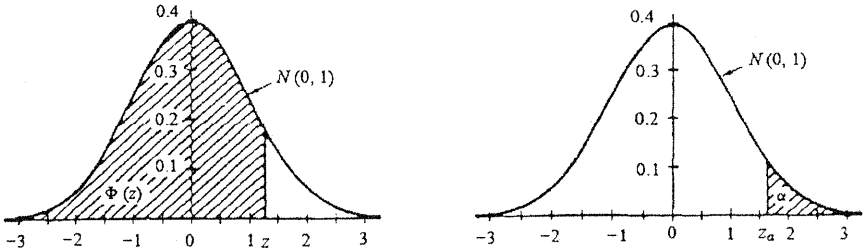
$$\underset{}{crime\ rate_i} = \underset{}{0.015} + \underset{(0.009)}{0.03} \cdot \underset{(0.02)}{alcohol}$$

where standard errors are shown in parentheses and below the estimated coefficients.

a. Does the data indicate that the crime rate tends to be higher in a region with higher average alcoholic beverage consumption? Find the confidence interval of the coefficient of *alcohol* and test at a significant level of 5%.
b. Interpret what *p*-value is and find the approximately *p*-value for the test in (a).

Appendix

Table I The Normal Distribution



$$P(Z \le z) = \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

$$[\Phi(-z) = 1 - \Phi(z)]$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

α	0.400	0.300	0.200	0.100	0.050	0.025	0.010	0.005	0.001
z_α	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.090
$z_{\alpha/2}$	0.842	1.036	1.282	1.645	1.960	2.240	2.576	2.807	3.291