

科目	微積分	適用系所	科技管理研究所	時間	100 分鐘
----	-----	------	---------	----	--------

※請務必在答案卷作答區內作答。 共 2 頁第 1 頁

一、填充題 (75%) [每一題 5 分，請依序填下答案於答案卷上，勿列出計算過程]

1. If $z = f(x, y) = 200 - 0.005x^2 - 0.01y^2$, find a vector giving the direction in which $f(x, y)$ decreases quickest at the point $(x, y, z) = (100, 50, 125)$: _____.

2. $\frac{\partial^3}{\partial x \partial y \partial y} \left(\frac{x^2 y^3 + 2xy^4}{12} \right) =$ _____.

3. The equation of the tangent line to the graph of $y = x^e + e^x$ at $x=1$ is _____.

4. The slope of the tangent line to the graph of $y = (x+2)^{1/x}$ at $x=1$ is _____.

5. By using the linearization of $f(x) = \sqrt[3]{x}$ at $x=32$ to approximate $\sqrt[3]{31.8}$, we have $\sqrt[3]{31.8} \doteq$ _____.

6. The vertical asymptote(s) of the graph of $f(x) = \frac{6x - 2x^2}{x^2 - 4x + 3}$ is (are) _____.

7. The horizontal asymptote(s) of the graph of $f(x) = \frac{2x}{\sqrt{x^2 + 3}}$ is (are) _____.

8. If $xy^2 - x^2y = 2$, then $\frac{dy}{dx}$ at the point $(x, y) = (1, -1)$ is _____.

9. $\int \frac{\sqrt[3]{\ln x}}{x} dx =$ _____.

10. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx =$ _____.

11. $\int x^2 \ln x dx =$ _____.

12. $\int_0^1 \int_y^1 2 \cdot e^{(x^2)} dx dy =$ _____.

13. The third-degree Maclaurin polynomial for $f(x) = e^{2x}$ is _____.

14. The limit of the sequence $\left\{ \frac{n^{100}}{2^{n/2}} \right\}$ is _____.

15. $\lim_{x \rightarrow \infty} \left(\frac{\ln x}{100\sqrt{x}} \right) =$ _____.

二、計算題 (25%) [各為 5 分, 10 分, 10 分; 請寫下詳細推導、計算過程, 否則不予計分]

1. Show that $\int_1^e \ln x \, dx = \int_1^e \frac{1}{x} \, dx$

2. Sketch and indicate the region bounded by the graphs of $y = x/2$ and $x = y^2 - 3$. Then, express the area of the region in terms of (1) integration with respect to x , and (2) integration with respect to y . (首先, 約略的畫出並標明此區域。然後, 把此區域的面積分別用對 x 的積分式, 及對 y 的積分式表示出來; 不必解出積分值。)

3. Sketch the graph of the function $f(x) = 3x^4 + 4x^3 + 1$, by discussing its increasing/decreasing, local extrema, concavity, and asymptotes. (由函數的遞增遞減、局部極值、凹凸性、及漸近線, 來畫出此函數的圖形。)