

逢甲大學101學年度碩士班招生考試試題 編號：024 科目代碼：

科目	統計學(含數理統計學)	適用系所	統計學系統計與精算碩士班應用統計暨計量財務組、精算組	時間	100 分鐘
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※請務必在答案卷作答區內作答。

共 2 頁第 1 頁

一、(30%) 請選出正確的答案 (共 10 小題，每小題 3 分)

- The sampling distribution of the sample means
 - is the probability distribution showing all possible values of the sample mean.
 - is used as a point estimator of the population mean μ .
 - is an unbiased estimator.
 - shows the distribution of all possible values of μ .
- A theorem that allows us to use the normal probability distribution to approximate the sampling distribution of sample means and sample proportions whenever the sample size is large is known as the
 - approximation theorem.
 - normal probability theorem.
 - central limit theorem.
 - central normality theorem.
- The weight of football players is normally distributed with a mean of 200 pounds and a standard deviation of 25 pounds. What percent of players weigh between 180 and 220 pounds?
 - 28.81%
 - 0.5762%
 - 0.281%
 - 57.62%
- Which of the following statements about a discrete random variable and its probability distribution are true?
 - Values of the random variable can never be negative.
 - Some negative values of $f(x)$ are allowed.
 - Values of $f(x)$ must be greater than or equal to zero.
 - The values of $f(x)$ increase to a maximum point and then decrease.
- After computing a confidence interval, the user believes the results are meaningless because the width of the interval is too large. Which one of the following is the best recommendation?
 - Increase the level of confidence for the interval.
 - Increase the sample size.
 - Decrease the sample size.
 - Reduce the population variance.
- The error of rejecting a true null hypothesis is
 - a Type I error.
 - a Type II error.
 - is the same as β .
 - committed when not enough information is available.
- If two independent large samples are taken from two populations, the sampling distribution of the difference between the two sample means
 - can be approximated by a Poisson distribution.
 - will have a variance of one.
 - can be approximated by a normal distribution.
 - will have a mean of one.
- The producer of a certain medicine claims that their bottling equipment is very accurate and that the standard deviation of all their filled bottles is 0.1 ounce or less. A sample of 20 bottles showed a standard deviation of 0.11. The test statistic to test the claim is
 - 400
 - 22.99
 - 4.85
 - 20
- The sampling distribution of the ratio of two independent sample variances taken from normal populations with equal variances is
 - an F distribution.
 - a chi-square distribution.
 - a t distribution.
 - a normal distribution.
- A regression analysis between sales (Y in \$1000) and advertising (X in dollars) resulted in the following equation

$$\hat{Y} = 30,000 + 4 X.$$

The above equation implies that an

- (A) increase of \$4 in advertising is associated with an increase of \$4,000 in sales.
 (B) increase of \$1 in advertising is associated with an increase of \$4,000 in sales.
 (C) increase of \$1 in advertising is associated with an increase of \$34,000 in sales.
 (D) increase of \$1 in advertising is associated with an increase of \$4 in sales.

二、(20%) 請選出正確的答案(共 5 小題，每小題 4 分)

1. The results of a recent poll on the preference of shoppers regarding two products are shown below.

<u>Shoppers Favoring</u>		
Product	Shoppers Surveyed	This Product
A	800	560
B	900	612

The 95% confidence interval estimate for the difference between the populations favoring the products is

- (A) 0.02 to 0.3 (B) 0.6 to 0.7 (C) 0.024 to 0.7 (D) -0.024 to 0.064
2. A regression and correlation analysis resulted in the following information regarding a dependent variable (y) and an independent variable (x).
 $n = 10$, $\Sigma X = 90$, $\Sigma(Y - \bar{Y})(X - \bar{X}) = 466$, $\Sigma Y = 170$, $\Sigma(X - \bar{X})^2 = 234$, $\Sigma(Y - \bar{Y})^2 = 1434$,
 $SSE = 505.98$. The least squares estimate of b_1 and the sample correlation coefficient are
 (A) -0.923; -0.8045 (B) 1.911; 0.8045 (C) 1.911; -0.8045 (D) 0.923; 0.8045
3. The life expectancy in the United States is 75 with a standard deviation of 7 years. A random sample of 49 individuals is selected. What is the probability that the sample mean will be larger than 77 years?
 (A) 0.0107 (B) 0.0321 (C) 0.0228 (D) 0.0438
4. A random sample of 144 observations has a mean of 20, a median of 21, and a mode of 22. The population standard deviation is known to equal 4.8. The 95.44% confidence interval for the population mean is
 (A) 15.2 to 24.8 (B) 19.200 to 20.800 (C) 19.216 to 20.784 (D) 21.2 to 22.8
5. In a recent survey in a Statistics class, it was determined that only 60% of the students attend class on Fridays. From past data it was noted that 98% of those who went to class on Fridays pass the course, while only 20% of those who did not go to class on Fridays passed the course. Given that a person passes the course, what is the probability that he/she attended classes on Fridays?
 (A) 0.88 (B) 0.668 (C) 0.12 (D) 0.332

三、(15%) If X and Y are any two random variables, then show that

- (a) $E(X) = E[E(X|Y)]$; and
 (b) $\text{Var}(X) = E[\text{Var}(X|Y)] + \text{Var}(E(X|Y))$.
 (c) For hierarchical model

$$Y|\Lambda \sim \text{Poisson}(\Lambda) \text{ and } \Lambda \sim \text{gamma}(\alpha, \beta)$$

find the mean and variance of Y .

四、(10%) Prove that if two events A and B are independent, so are A and B^c , A^c and B , and A^c and B^c .

五、(15%) Let X_1, X_2, \dots, X_n be independent and identically distributed $\mathcal{N}(\mu, \sigma^2)$, where both μ and σ^2 are unknown.

- (a) Find the maximum likelihood estimators (MLEs) for μ and σ^2 .
 (b) Are the MLEs for μ and σ^2 derived in part (a) unbiased? Explain.
 (c) Develop a likelihood ratio test for the hypothesis testing:

$$H_0: \mu = \mu_0 \text{ versus } H_1: \mu \neq \mu_0.$$

六、(10%) Let X and Y be two random variables with a joint probability density function as

$$f(x, y) = \begin{cases} 8xy, & 0 \leq x \leq y \leq 1; \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Are X and Y independent?
 (b) Derive the conditional probability distribution function of X given Y , say $F_{X|Y}(x|y)$.