

科目	微積分	適用系所	統計學系統計與精算碩士班應用統計暨計量財務組、精算組	時間	100 分鐘
----	-----	------	----------------------------	----	--------

※請務必在答案卷作答區內作答。

- Suppose that $f'(x) > 0$ and $g'(x) > 0$ for all x . What simple additional conditions (if any) are needed to guarantee that:
 - (8%) $f(x) \cdot g(x)$ is increasing for all x ;
 - (8%) $f(g(x))$ is increasing for all x ?
- Please show the details to solve the following questions:
 - (8%) Evaluate $\int_0^1 [x^2 + x(x^2 + 1)^4] dx$.
 - (8%) Let $y = f(x) = x^5 + 2x + 1$. Compute $(f^{-1})'(4)$.
- (10%) Show that every absolutely convergent series is convergent.
- (8%) Show that the sequence $\{a_n\}_{n=1}^{\infty}$ converges, and find its limit, where $a_1 = 1$ and $a_{n+1} = (2 + a_n)^{1/2}$, $n = 1, 2, \dots$.
- (10%) Suppose that the f is integrable on $[a, b]$ and $m \leq f \leq M$ for all x in $[a, b]$. Please find the upper and lower bound of $\int_a^b f(x) dx$.
- (10%) Find $D_x \sin[\cos(x^2)]$.
- (15%) Evaluate $\int_0^{\sqrt{\pi}/2} x \sin^3(x^2) \cos(x^2) dx$.
- (15%) Find $\frac{dy}{dx}$, if $y = \cos^3(x^2 + 1)$.