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| 科目 | 工程數學 | 適用系所 | 航太與系統工程學系固力組、熱流組、控制組 | 時間 | 100 分鐘 |
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※請務必在答案卷作答區內作答。 共 1 頁第 1 頁

1. If $\vec{V}(x, y) = e^{xy}\vec{i} + (x - y)\vec{j} + x \sin y\vec{k}$, please find $\frac{\partial \vec{V}}{\partial x} \times \frac{\partial \vec{V}}{\partial y}$. (15%)

2. Please solve the following differential equation. (15%)

$$(1 + 2e^{x/y})dx + 2e^{x/y}(1 - x/y)dy = 0$$

3. Please use the Laplace Transform to solve the following differential equation. (20%)

$$y'' - 3y' + 2y = 4e^{2x}, \quad y(0) = -3, \quad y'(0) = 5.$$

4. Find the eigenvalues and eigenvectors of the matrix $[A]$.

Also, diagonalize the following matrix $[A]$. Show the details. (15%)

$$[A] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

5. Using Green's theorem, evaluate $\int_C \vec{F}(\vec{r}) \cdot d\vec{r}$ counterclockwise around the boundary curve

C of the region R , where $\vec{F} = (y^2 - 7y)\vec{i} + (2xy + 2x)\vec{j}$ and $R: x^2 + y^2 = 1$. (20%)

6. If u and v are harmonic functions, show that

$$\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) + i \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$

is an analytic function. (15%)