東海大學 101 學年度碩士班招生入學考試試題

考試科目：線性代數A　　　　　　應考系所：應數系

本試題共 1 頁：第 1 頁　　　（如有缺損或印刷不清者，應即舉手請監試人員處理）

一、 計算證明題:(要有計算過程，否則不予給分)　　　　(6% + 2% + 4%)

Let $V = \mathbb{R}^3$, and $W = \{(x, y, z)|x + 3y - 2z = 0\}$ be a subspace of V

(1) Find an orthonormal basis(標準正交基底) for W

(2) Find $W^{\perp}$(the orthogonal complement of W (W的正交補集))

(3) Let $\vec{u} = (2, 1, 3)$, find $Min_{\vec{v} \in W}\|\vec{v} - \vec{u}\|$

二、 計算證明題:(要有計算過程，否則不予給分)　　　　(6% + 8% + 20%)

Let $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

(1) Consider the system $A\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. By Cramer's Rule(克拉瑪公式), find $\vec{x}$.

(2) Express A as a product of elementary matrices(基本矩陣的乘積).

(3) Find an invertible matrix(可逆矩陣) $P \in M_{3\times3}(\mathbb{R})$ and a diagonal(對角矩陣) matrix $D \in M_{3\times3}(\mathbb{R})$ such that $A = P \cdot D \cdot P^{-1}$.

三、 True or False:(對或錯。對的，請先回答"True"，即給2分，並證明，則再給4分；錯的，請先回答"False"，即給2分，並舉反例，則再給4分。) Let V and W be vector spaces over the field F. Let $F = \mathbb{R}$ or $\mathbb{C}$.　　　　$(9 \times 2\% + 9 \times 4\%)$

(1) The function det: $M_{n\times n}(F) \longrightarrow F$ is a linear transformation(線性轉換).

(2) Let A and B be $n \times n$ matrices. Then $(A + B)(A - B) = A^2 - B^2$

(3) An $m \times n$ linear system has a unique solution(唯一解), infinitely many solutions(無限多組解), or no solutions(無解).

(4) $\mathbb{C}$ is a 2-dimensional(2維) vector space over $\mathbb{R}$.

(5) The union(聯集) of two subspaces of a vector space is a subspace(子空間).

(6) Given $\vec{v_1}$, $\vec{v_2} \in V$ and $\vec{w_1}$, $\vec{w_2} \in W$, then there exists a linear transformation $T : V \longrightarrow W$ such that $T(\vec{v_1}) = (\vec{w_1})$ and $T(\vec{v_2}) = (\vec{w_2})$

(7) Let $T : V \longrightarrow W$ be a function. If $T(\vec{0_v}) \neq \vec{0_w}$, then T is not linear.

(8) Let A be an $n \times n$ matrix. Then A and A^t have the same eigenvalues.

(9) Let $T : V \longrightarrow W$ be a linear. If λ is an eigenvalue of T, then the eigenspace(特徵空間) $E_\lambda = \{\vec{v} \in V|T(\vec{v} = \lambda\vec{v}\}$ is a subspace of V.