元智大學 101 學年度研究所 **碩士班** 招生試題卷

系(所)別：通訊工程學系碩士班　組別：通訊組　科目：工程數學　用紙第 1 頁共 2 頁

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1. The sub-experiment A contains four outcomes $\alpha, \beta, \gamma, \omega$. Now suppose the experiment B consists of two independent repetitions of the sub-experiment A. (25%)

(a) Give one example of event spaces of the sub-experiment A. (5%)

(b) What is the sample space of the experiment B? (5%)

(c) Design a random variable for the experiment B. (5%)

(d) Design a question related to the experiment B. The question should be solved by 'Law of total probability'. You should clearly assign the probabilities and calculate the solution. (10%)

2. The probability density function of a random variable X is (25%)

$$f_X(x) = \begin{cases} kxe^{-x/2}, & x \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the constant k. (5%)

(b) What is $P[-1 \le X < 2]$? (5%)

(c) Let $H_X(x) = P[X < x]$. Please express $H_X(x)$. (15%)

3. Let the equation of the parabola be $y = a_0 + a_1 x + a_2 x^2$. Please determine the coefficients (a_0, a_1, a_2) to approximate the following set of measurement data

	i=1	i=2	i=3	i=4
x_i	-1	0	0	1
y_i	0	3	-1	0

and get the minimum value $\sum_{i=1}^{4}\{y_i - (a_0 + a_1 x_i + a_2 x_i^2)\}^2$. (10%)

4. Let **A**, **B**, **C**, and **D** be $n\times n$, $n\times k$, $k\times k$, and $k\times n$ matrices, respectively. (7%)

Given the matrix inversion lemma:

$$(\mathbf{A}+\mathbf{BCD})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}\left(\mathbf{DA}^{-1}\mathbf{B}+\mathbf{C}^{-1}\right)^{-1}\mathbf{DA}^{-1}$$

where **C**, **A**, and $(\mathbf{A}+\mathbf{BCD})$ are invertible.

Please show the special case:

$$\left(\mathbf{A}+\mathbf{uv}^H\right)^{-1} = \mathbf{A}^{-1} - \frac{\mathbf{A}^{-1}\mathbf{uv}^H\mathbf{A}^{-1}}{1+\mathbf{v}^H\mathbf{A}^{-1}\mathbf{u}}$$

where **u** and **v** are $n\times 1$ matrices, and $\left(1+\mathbf{v}^H\mathbf{A}^{-1}\mathbf{u}\right) \neq 0$.

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5. Consider the nonhomogeneous linear system: (7%)

$$\mathbf{A}\mathbf{x} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

Let the solution be $\mathbf{x} = r\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ with r being arbitrary value.

Please determine the transformation matrix $\mathbf{A}$=?

6. Consider the linear transformation T: $\mathbf{R}^3 \to \mathbf{R}^3$. (16%)

$$T(x, y, z) = (x + 3y + 2z, y + z, x + 2y + z)$$

(a) Find the kernel of T, ker(T). (4%)

(b) Find the range of T, range(T). (4%)

(c) Show that dim ker(T) + dim range(T) = dim domain(T). (4%)

(d) Find the set of vectors that are mapped by T into the vector (1,1,0). Is this set a subspace? (4%)

7. Let $\mathbf{A} = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$.

Please find an unitary matrix $\mathbf{P}$ and an upper triangular matrix $\mathbf{R}$ so that $\mathbf{P}^T\mathbf{A}\mathbf{P} = \mathbf{R}$. (10%)